Internal dynamics of globular clusters

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Summary. Galactic globular clusters, which are ancient building blocks of our Galaxy, represent a very interesting family of stellar systems in which some fundamental dynamical processes have taken place on time scales shorter than the age of the universe. In contrast with galaxies, these clusters represent unique laboratories for learning about two-body relaxation, mass segregation from equipartition of energy, stellar collisions, stellar mergers, and core collapse. In the present review, we summarize the tremendous developments, as much theoretical as observational, that have taken place during the last two decades, and which have led to a quantum jump in our understanding of these beautiful dynamical systems.

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1. Introduction

Till the late ninety seventies, globular clusters were thought to be relatively static stellar systems. This was partly due to the fact that most observed surface-brightness profiles of globular clusters (obtained from aperture photometry in the central and intermediate parts, and star counts in the outer parts) were successfully fitted by equilibrium models. Some of these models are based on lowered maxwellians and commonly known as King models (King 1966); they are the simplest dynamical models which incorporate the three most important elements governing globular cluster structure: dynamical equilibrium, two-body relaxation, and tidal truncation.

It had been already known, since the early sixties, that globular clusters had to evolve dynamically, even when considering only relaxation, which causes stars to escape, consequently cluster cores to contract and envelopes to expand. But dynamical evolution of globular clusters was not yet a field of research by itself, since the very few theoretical investigations had led to a most puzzling paradox: core collapse (Hénon 1961, Lynden-Bell & Wood 1968, Larson 1970a,b, Lynden-Bell & Eggleton 1980). It was only in the early eighties that the field grew dramatically. On the theoretical side, the development of high-speed computers allowed numerical simulations of dynamical evolution. Nowadays, Fokker-Planck and conducting-gas-sphere evolutionary models have been computed well into core collapse and beyond, leading to the discovery of

possible post-collapse oscillations. In a similar way, hardware and software improvements of N-body codes provide very interesting first results for 10^4-body simulations (Makino 1996a,b, Spurzem & Aarseth 1996), and give the first genuine hope, in a few years, for 10^5-body simulations. On the observational side, the manufacture of low-readout-noise Charge Coupled Devices (CCDs), combined since 1990 with the high spatial resolution of the Hubble Space Telescope (HST), allow long integrations on faint astronomical targets in crowded fields, and provide improved data analyzed with sophisticated software packages.

It is not an exaggeration to say that our vision of globular cluster dynamics has significantly been altered during the last decade. Globular clusters are not dormant stellar systems. Their apparent smoothness, regularity, and symmetry are hiding everything but simplicity. Because of typical individual masses of a few $10^5 M_\odot$, intermediate between open clusters and dwarf galaxies, globular clusters are of crucial importance in stellar dynamics: fundamental dynamical processes (such as relaxation, mass segregation, core collapse) take place in these systems on time scales shorter than the Hubble time. Recent theoretical and observational studies of high-concentration globular clusters, with $c = \log\,(r_t/r_c) \gtrsim 2$, where r_t and r_c are the tidal and core radii, have confirmed what was strongly suspected: stellar and dynamical evolutions are intimately connected. Observational studies concerning individual stars as well as those devoted to integrated properties of stellar distributions (e.g., color and population gradients) show that stellar encounters, collisions, and mergers complicate and enrich the dynamical study of globular clusters.

In this review we describe the present status of our knowledge of the internal dynamics of globular clusters, from both theoretical and observational points of view. It is structured as follows:

Section 2 gives a tentative definition of globular clusters;

Section 3 gives a brief historical summary of the study of globular cluster dynamics;

Section 4 describes the general characteristics of the globular clusters in our Galaxy and discusses a few astrophysical properties of this cluster system;

Section 5 summarizes what is known (and above all unknown) about the formation of globular clusters;

Section 6 describes the different kinds of observations providing dynamical constraints;

Section 7 describes clusters in terms of quasi-static equilibrium, i.e., especially in the pre-collapse regime;

Section 8 describes the different kinds of evolutionary models;

Section 9 describes the evolution towards catastrophic phases, provides the existing observational evidence for core collapse, and discusses the influence on stellar populations of the high stellar density resulting from dynamical evolution;

Section 10 describes the late phases of the evolution and disruption; finally,

Section 11 discusses possible future directions, from both theoretical and observational points of view.

Hereafter follows, for the interested reader, a nonexhaustive list of some of the most important and already published reviews, monographs, and proceedings related to the dynamics of globular clusters.

First, two extensive reviews about dynamical evolution and binaries in globular clusters, respectively:

- Lightman, A.P., Shapiro, S.L., 1978, *Dynamical Evolution of Globular Clusters*, Rev. Mod. Phys., 50, 437;
- Hut, P., McMillan, S.L.W., Goodman, J., Mateo, M., Phinney, E.S., Pryor, C., Richer, H.B., Verbunt, F., Weinberg, M., 1992, *Binaries in Globular Clusters*, PASP, 104, 981.

Second, seven reviews published in Annual Review Astronomy & Astrophysics and related to globular cluster dynamics:

- Michie, R.W., 1964, *The Dynamics of Star Clusters,* ARA&A, 2, 49;
- Harris, W.E., Racine, R., 1979, *Globular Clusters in Galaxies,* ARA&A, 17, 241;
- Freeman, K.C., Norris, J., 1981, *The Chemical Composition, Structure, and Dynamics of Globular Clusters*, ARA&A, 19, 319;
- Elson, R.A.W., Hut, P., Inagaki, S., 1987, *Dynamical Evolution of Globular Clusters,* ARA&A, 25, 565;
- Valtonen, M., Mikkola, S., 1991, *The Few-Body Problem in Astrophysics,* ARA&A, 29, 9;
- Harris, W.E., 1991, *Globular Cluster Systems in Galaxies Beyond the Local Group,* ARA&A, 29, 543;
- Bailyn, C.D., 1995, *Binary Stars, Blue Stragglers, and the Dynamical Evolution of Globular Clusters,* ARA&A, 33, 133.

Third, three fundamental books:

- Saslaw, W.C., 1987, *Gravitational Physics of Stellar and Galactic Systems,* (Cambridge: Cambridge University Press);
- Spitzer, L., 1987, *Dynamical Evolution of Globular Clusters,* (Princeton: Princeton University Press);
- Binney, J., Tremaine. S., 1987, *Galactic Dynamics,* (Princeton: Princeton University Press).

Fourth, the proceedings of thirteen workshops and conferences related to globular cluster dynamics, all of them containing excellent reviews:

- Hayli, A., ed., 1975, *Dynamics of Stellar Systems*, IAU Symp. 69, (Dordrecht: Reidel);

- Hesser, J.E., ed., 1980, *Star Clusters*, IAU Symp. 85, (Dordrecht: Reidel);
- Goodman, J., Hut, P., eds., 1985, *Dynamics of Star Clusters*, IAU Symp. 113, (Dordrecht: Reidel);
- de Zeeuw, T., ed., 1987, *Structure and Dynamics of Elliptical Galaxies*, IAU Symp. 127, (Dordrecht: Reidel);
- Grindlay, J.E., Philip, A.G.D., eds., 1988, *The Harlow-Shapley Symposium on Globular Cluster Systems in Galaxies*, IAU Symp. 126, (Dordrecht: Kluwer);
- Merritt, D., ed., 1989, *Dynamics of Dense Stellar Systems*, (Cambridge: Cambridge University Press);
- Valtonen M.J., ed., 1988, *The Few Body Problem*, IAU Coll. 96., (Dordrecht: Kluwer);
- Janes, K., ed., 1991, *The Formation and Evolution of Star Clusters*, ASP Conference Series, Vol. 13, (San Francisco: ASP);
- Smith, G.H., Brodie, J.P., eds., 1993, *The Globular Cluster - Galaxy Connection*, ASP Conference Series, Vol. 48, (San Francisco: ASP);
- Djorgovski, S.G., Meylan, G., eds., 1993, *Structure and Dynamics of Globular Clusters*, ASP Conference Series, Vol. 50, (San Francisco: ASP);
- Saffer, R.A., ed., 1993, *Blue Stragglers*, ASP Conference Series, Vol. 53, (San Francisco: ASP);
- Milone E.F., Mermilliod J.-C., eds., 1996, *The Origins, Evolution, and Destinies of Binary Stars in Clusters*, ASP Conference Series, Vol. 90, (San Francisco: ASP);
- Hut P., Makino J., eds., 1996, *Dynamical Evolution of Star Clusters: Confrontation of Theory and Observation*, IAU Symp. 174, (Dordrecht: Kluwer).

In addition, some articles and extensive lists of references are found in the triennial Transactions of the International Astronomical Union, *Reports on Astronomy*.

2. Definition of globular clusters

The usual definition of a globular cluster describes it as an old star cluster (with an age τ larger than about 10 Gyr) found in the bulge and halo regions of the Galaxy. A precise determination of the absolute age of the oldest galactic globular clusters is still an elusive cosmological problem. From both observational and theoretical arguments, Walker (1992) and Chaboyer (1995) reach a similar conclusion: the absolute ages of the oldest globular clusters are found to lie in the range 11-21 Gyr. A mean age of about $\tau \sim 15$ Gyr is generally accepted, but calibrations through stellar evolution models are uncertain: e.g., Shi et al. (1995) and Shi (1995) shows that adopting an initial helium abundance of $Y =$

0.28 or a mass loss rate $\dot{M} \sim 10^{-11}\ M_\odot\ \mathrm{yr}^{-1}$ near the main sequence turn-off region lowers the current age estimate from 15 Gyr to about 10-12 Gyr. See also Mazzitelli et al. (1995) for an investigation of globular cluster ages with updated input physics and van den Bergh (1995a,b,c), Sarajedini et al. (1995), and Chaboyer et al. (1996a,b) for interesting discussions.

Contrary to absolute ages, the relative ages of some galactic globular clusters are more precisely known. They are obtained by comparison of their color-magnitude diagrams, which display clear differences in age of about 3 Gyr (Bolte 1989). Chaboyer et al. (1996c), on new age estimates for 43 globular clusters, argue that their sample has a statistically significant age spread of at least 5 Gyr.

The above age definition ($\tau \gtrsim 10$ Gyr) suits the kind of globular clusters which are the main subject of the present review. Nevertheless, other galaxies contain younger stars clusters among which some may be the progenitors of stellar clusters similar to the galactic globular clusters.

It is also worth mentioning that, already in our Galaxy, globular clusters differ strongly from one to the other, e.g., in integrated absolute magnitude and total mass, which range from $M_V^{int} = -10.1$ and $M_{tot} = 5 \times 10^6 M_\odot$ (Meylan et al. 1994, 1995) for the giant galactic globular cluster ω Centauri down to $M_V^{int} = -1.7$ and $M_{tot} \simeq 10^3 M_\odot$ for the Lilliputian galactic globular cluster AM-4 (Inman & Carney 1987). AM-4 is located at $\simeq 26$ kpc from the galactic centre, and at $\simeq 17$ kpc above the galactic plane and cannot be considered to be an old open cluster. The uncertainties on the above total mass estimates, perhaps as large as 100%, do not alter the fact that, in our Galaxy, the individual masses of globular clusters range over three orders of magnitude. It is not known to what extent these mass differences are "congenital" or due to subsequent pruning by dynamical evolution.

Although most galactic globular clusters are located within 20 kpc from the galactic centre, it is worth mentioning the existence of a few very remote galactic clusters. The distance record is held by AM-1 (Aaronson et al. 1984, Madore & Freedman 1989) which is located at about 120 kpc from the galactic centre, i.e., more than twice the distance to the Large Magellanic Cloud.

The most recent update of observational and structural parameters of all known galactic globular clusters may be found in the appendices and tables of the proceedings of the 1992 Berkeley workshop edited by Djorgovski & Meylan (1993).

Globular clusters are observed in other galaxies of the Local Group and beyond (cf. Harris 1991 and references therein for globular cluster systems in galaxies). The major difference with the galactic globular clusters resides in the fact that the above definition based on the age only ($\tau \gtrsim 10$ Gyr) is no longer sufficient. Rich stellar systems with ages smaller than 10 Gyr are observed. E.g., in the Magellanic Clouds, the two dwarf irregular companion galaxies of ours, there are star clusters with ages $10^6 \lesssim \tau \lesssim 10^9$ yr. There is still debate about the status of the richest of these star clusters: are they the progenitors of genuine old globular clusters? Should the previous definition, related to age only, be relaxed in order to include, e.g., clusters of different ages (a car is

called a car, independently of the fact that it is a new or used one)? Following van den Bergh (1993d), the most powerful discriminant between open and globular clusters is their luminosity function: the globular clusters have a gaussian luminosity function whereas the open clusters have a luminosity function increasing monotonically towards faint luminosities. Is this discriminant totally independent of the definitions adopted for sorting between open and globular clusters? See recent interesting discussions by van den Bergh (1995a,b,c).

Not every globular cluster has a mass of about $10^6 M_\odot$. Not every globular cluster has an age larger than 10 Gyr. Consequently, there is no simple (one-parameter) definition of globular clusters which would apply to every globular cluster around any galaxy. In a few cases, the classification between globular and open clusters remains unclear (e.g., see Ortolani et al. 1995). The discussion about a clear definition of globular/open clusters may look semantic after all, but it becomes essential when, e.g., luminosity functions of systems of globular and open clusters are used for constraining the importance of galaxy mergers in cluster formation (see, e.g., van den Bergh 1995b, 1996 and references therein). Fortunately, in the framework of the present review, a perfect definition of globular clusters is not essential since an overwhelming fraction of the high-quality dynamical observations of globular clusters concerns only the nearby rich (M_{tot} equal a few $10^5 M_\odot$) galactic globular clusters. All these observed clusters have ages τ larger than about 10 Gyr, and their large numbers of member stars make them interesting from a dynamical point of view. It is to these observations that theoretical models are fitted, and it is against these stellar systems that our theoretical understanding of the internal dynamics of globular clusters is tested.

3. Internal dynamics: a brief historical summary

Apart from the catalog of Charles Messier (1784), which mentions 28 galactic globular clusters visible from Europe, the first scientific description of globular clusters — which clearly identified them as huge swarms of stars of regular symmetrical appearance — was published by William Herschel (1814). A few decades later, this work was extended to the southern hemisphere by his son John Herschel (1847).

Mere descriptions of visual observations were superseded gradually, during the second half of the 19^{th} century, by more useful and efficient observations thanks to the development, and numerous sophisticated improvements till not long ago, of photographic techniques applied to astronomy. It is from photographic observations of two globular clusters — ω Centauri and 47 Tucanae — that Bailey (1893) made what were probably the first extensive star counts, which represent the oldest observational constraint for the study of globular clusters. Bailey's counts added to some new material concerning other clusters were used by Pickering (1897) in the first important comparisons between observed and theoretical profiles in order to study the radial distribution of stars

in clusters. A few years later, W.E. Plummer (1905) and von Zeipel (1908) showed, in studies of M3, M13, 47 Tucanae, and ω Centauri, how the radial space distribution of stars may be deduced numerically from the observed projected density profile. Von Zeipel compared these profiles with those to be expected for a spherical mass of gas in isothermal equilibrium. At most a little physics was present in such studies.

In parallel with the improvement of observational techniques, the second half of the 19$^{\text{th}}$ century experienced also dramatic progress in theoretical physics, with the invention of the new fields of thermodynamics and statistical mechanics, in order to describe gases with molecules of infinitesimal size. Maxwell (1860) wrote down the now famous maxwellian law of the distribution of velocities in a work which gave birth to the kinetic theory of gases, and it was further developed by Boltzmann (1896), among others.

In the early years of the 20$^{\text{th}}$ century, some parallels were drawn between a molecular gas and star clusters: the stars were considered as mass points representing the molecules in a collisionless gas. The analogy between a gas of molecules and a gas of stars is subject to criticisms, since the mean free path of a molecule is generally quite small compared with the size or scale height of the system, whereas the mean free path of a star is much larger then the diameter of the cluster; in addition molecules travel along straight lines, while stars move along orbits in the gravitational potential of all the other stars of the stellar system. Stellar collisions in clusters were studied by Jeans (1913), who remarked that they might be important in such stellar systems. The problem was then to seek the possible spherical distribution of such a gas in a steady state. H.C. Plummer (1911, 1915) pursued the search for a physical basis on which the distribution of stars in globular clusters could be established, a search followed by a flurry of essential theoretical contributions by Eddington (1913, 1915a, 1915b, 1916) and by Jeans (1913, 1915, 1916a, 1916b).

This amazing burst of fundamental papers was followed by a relatively dormant period which ended with another major era in cluster theory, containing the essential theoretical contributions by Ambartsumian (1938), Spitzer (1940), and Chandrasekhar (1942, his §5.8), who investigated the consequences of stellar encounters. The next burst of fundamental papers took place in the late fifties and early sixties, with the contributions by King (1958b, 1962, 1965, 1966) and by Michie (1961, 1963a,b,c,d), among others. At that time, two clusters, namely, M3 and ω Centauri, were the subjects of studies by Oort & van Herk (1959) and Dickens & Woolley (1967), respectively. These two papers initiated the modern interplay of observation and model-building that still continues today. The paper by Gunn & Griffin (1979) was another notable landmark in these developments.

Already before, and also after, the pioneering work of von Hoerner (1960), who made the first N-body calculations with $N = 16$, it was realized that computation of individual stellar motions could be replaced by statistical methods. The structure of a globular cluster is defined at each moment by a distribution function in a phase space with 7 dimensions (positions, velocities, and time). Unfortunately, the numerical study of such a general form is in-

tractable. It is necessary to make some simplifying hypotheses, e.g., spherical symmetry of the cluster, or quasi-static equilibrium. The major simplification consists in considering separately the problems of structure and evolution.

A series of works had studied the structure of globular clusters without taking into account their evolution (e.g., Plummer 1911, Eddington 1915a,b, 1916, Jeans 1915, 1916a,b, Chandrasekhar 1942, Camm 1952, Woolley & Robertson 1956, Hénon 1959, Michie 1963a, King 1966). Depending on further simplifications varying from one author to another, the distribution function may have any form, as long as it satisfies Jeans' theorem. King's studies (1966) have shown "lowered maxwellian" energy dependence to be a good approximation to the solution of the Fokker-Planck equation describing the phase-space diffusion and evaporation of stellar systems like globular clusters. These models fit the density profiles of globular clusters rather well. Nevertheless, this agreement need have no deep physical meaning, given the ad hoc hypotheses simplifying the fundamental equations, even if there is some dynamical justification for King's choice.

On the contrary, other studies had considered the evolution of globular clusters with a fixed structure (e.g., Spitzer 1940, Chandrasekhar 1943a,b,c, King 1958a,b,c, Spitzer & Härm 1958, von Hoerner 1958, Agekian 1958, Hénon 1960a, King 1960, Michie 1961). In most cases, the cluster was supposed to be homogeneous with a uniform gravitational potential. The results obtained — essentially the escape rate of stars from the clusters — were rather different, once again because of simplifying hypotheses.

In reality, structure and evolution cannot be dissociated: they are intimately linked, determined the one by the other. Hénon (1961, 1965) made the first attempt to solve the structure and evolution equations simultaneously, in the simplified case of a self-similar evolution with a distribution function depending only on the total energy (isotropy of the velocity dispersion) and with all stars having the same mass. Even to other theorists his model looked pathological: it had infinite central density and a flux of energy emerged from the central singularity. But Hénon showed that a cluster *without* such a singularity would evolve into one that did, and he realised that, in a real system, the flux of energy might well be supplied by the formation and evolution of binary stars. Hénon was right, and his results had given him a first glance at what was to become the Holy Grail of globular cluster dynamics: core collapse.

The whole concept of core collapse, linked to the gravothermal instability which may develop in a gravitational system because of its negative specific heat, was first investigated by Antonov (1962), and Lynden-Bell & Wood (1968). What was then called the gravothermal catastrophe was eventually recognized as being not so catastrophic after all, since the cluster core does not collapse for ever but bounces back towards lower stellar density phases. Again it was Hénon, this time in his 1975 paper, who showed theorists the way past the apparent impasse of core collapse into the post-collapse phase of evolution.

This brief history is far from being exhaustive but brings us to the seventies. It is the further modern theoretical and observational developments which are the subject of the present review.

4. Characteristics of the globular clusters in our Galaxy

After having provided observational indications of the extended structure of our Galaxy and the eccentric position of the sun with respect to the galactic centre (Shapley 1930), the globular cluster system of our Galaxy has been long recognized as an interesting tool to study the early dynamical and chemical evolution of the galactic halo (Trumpler 1930). Over the last few decades, analyses have tried to show evidence of a metal-rich disk subsystem, complementary to the metal-poor halo subsystem (e.g., Baade 1958, Kinman 1959, Morgan 1959, Woltjer 1975, Harris 1976, Hartwick & Sargent 1978, van den Bergh 1979, Zinn 1980, Frenk & White 1980, 1982, Zinn 1985, Hesser et al. 1986, Armandroff & Zinn 1988, Armandroff 1988, Armandroff 1989, Thomas 1989, Minniti 1995). There is now clear evidence indicating that the globular cluster system of the Galaxy consists of two separate subsystems, a slowly rotating halo subsystem and a rapidly rotating disk subsystem. A detailed knowledge of these subsystems is essential for understanding the fact that some internal properties of clusters, e.g., the concentration parameter, correlate well with global variables such as the galactocentric distance. This suggests that some external effects strongly influence the internal dynamical evolution of a globular cluster (Chernoff & Djorgovski 1989).

Fig. 4.1. Projected distribution of the 143 known globulars in galactic coordinates (from Djorgovski & Meylan 1994, Fig. 1). The symbol size scales with the logarithm of the luminosity. The strong central concentration is obvious.

The galactic globular system consists of 143 confirmed globular clusters (Djorgovski & Meylan 1993b). Fig. 4.1 shows the distribution of clusters on the galactic sky (Djorgovski & Meylan 1994). The well-known strong central concentration is the most obvious feature of the distribution. The absence of an obvious zone of avoidance near the galactic plane immediately suggests no large numbers of clusters are missing due to obscuration. Nevertheless, it

is very likely that some clusters are still missing, lost in the obscured areas near the galactic plane or in the outer parts of the halo. The latest addition to the list consists of the new globular cluster C J0907–372 (Pyxis) recently discovered by Weinberger (1995) and confirmed by Da Costa (1995) and Irwin et al. (1995). The kinematics and dynamics of the galactic globular cluster system have been studied by, e.g., Frenk & White (1980, 1982), Innanen et al. (1983), and Thomas (1989).

4.1 The radial distribution

It is possible to parameterize the radial distribution of the galactic globular clusters by using a simple power law with a core:

$$\rho(r) \ = \ \rho_0 \ \left(1 \ + \ \frac{r}{r_c}\right)^{-\alpha} \tag{4.1}$$

This approach is purely empirical, and it is not meant to imply any physical meaning of the distribution given by Eq. 4.1 (Djorgovski & Meylan 1994). Such a simple fit neglects the disk-halo dichotomy, and many other fine details. The purpose is simply to estimate the number of clusters which may be missing in the central parts of the Galaxy.

Taking into account the probable incompleteness of the data and the distance errors, which could be rather substantial for the heavily obscured clusters at small galactocentric radii, Djorgovski & Meylan (1994) perform fits to the data with model curves of various values of the core radius r_c and the power-law exponent α. Good matches are found for the values of $\alpha \sim$ 3.5-4. A generally quoted value in the literature is 3.5 (see, e.g., Harris 1976, Zinn 1985). The core radii $r_c \sim$ 0.5-2 kpc. The flattening of the distribution near the centre − into a core − is probably due to a combination of three effects: smearing due to the distance errors, genuine clusters which are missing due to obscuration, and the real flattening of the distribution. The latter may reflect the initial conditions, but also possible dynamical effects, viz., a more effective tidal destruction of clusters near the galactic centre. For the faint clusters towards the galactic centre, there is also the mere problem of classification: e.g., NGC 6540, previously considered as an open cluster, has been recently recognized as a globular cluster (Bica et al. 1994).

The steep observed slope of this distribution differs significantly from the density law of the dark halo, $\rho(r) \sim r^{-2}$, which results in a flat rotation curve. It is hard to imagine an evolutionary process which could convert, over an Hubble time, a r^{-2} distribution into a $r^{-3.5}$ one, for the globular clusters but not for the dark halo material. This implies a different origin for the globular cluster system (and presumably the visible stellar halo), and the dark halo, whatever its constituents are.

The apparent core radii found by Djorgovski & Meylan (1994) ($\sim 0.5 - 2$ kpc) are considerably larger than the characteristic radii for the stellar distri-

bution in the bulge: Blanco & Terndrup (1989) give $r_c = 0.11 \pm 0.04$ kpc for the bulge light. This is reminiscent of the situation seen in M87, where the core radius of the globular cluster system is some 13 times larger than that of the underlying galaxy's light (Lauer & Kormendy 1986).

The approach by Djorgovski & Meylan (1994) and other alternatives show that the number of missing clusters at low latitudes and/or near the galactic centre is perhaps of the order of 10. Similar conclusions have been reached by Racine & Harris (1989), who performed a more detailed analysis, and also by Woltjer (1975) and Oort (1977). In conclusion, there is probably a slight selection effect, leading to an incompleteness of $\sim 5\%$ of the total number of clusters.

4.2 The clusters away from, and near to, the galactic bulge

[Fe/H] values, compiled by Zinn & West (1984), Zinn (1985), and Armandroff & Zinn (1988), exist for 119 of the 143 galactic globular clusters. Since there is no clear spatial division between halo and disk populations, it is generally admitted, from the distribution of IRAS sources, that the galactic bulge extends to an angular distance ω of approximately 15° from the galactic centre, with ω = 15° being the dividing line between bulge and non-bulge regions (Zinn 1990, 1996). There is also no clear division between disk globular clusters and open clusters: from the color-magnitude diagram of Lyngå 7, Ortolani et al. (1993) and Tavarez & Friel (1995) observe that this cluster, previously classified as an open cluster, might be a metal rich globular cluster or, alternatively, the oldest open cluster so far detected.

The clusters away from the galactic bulge ($\omega > 15°$). The observational evidence (see, e.g., Fig. 1 in Zinn 1990) indicating that distinct halo and disk subsystems exist among the clusters with $\omega > 15°$ comes from:

(i) the distribution of cluster [Fe/H] values, which is bimodal with peaks at [Fe/H] = –1.6 and –0.6;

(ii) the distribution of distances from the galactic plane |Z|, which shows that while the metal-poor clusters are scattered over a large range in distance, the clusters more metal-rich than [Fe/H] = –1 are all at less than 4 kpc from the galactic plane;

(iii) the metal-poor and metal-rich clusters, which have very different values for the V_{rot}/σ_{los} ratio;

(iv) the most metal-rich bin, which has both the largest value of rotational velocity, $V_{rot} = 172 \pm 26$ km s^{-1}, and the smallest value of the line-of-sight velocity dispersion, $\sigma_{los} = 60 \pm 14$ km s^{-1}.

The clusters near the galactic bulge ($\omega < 15°$). The distribution of cluster [Fe/H] values is also bimodal, with two peaks at approximately the same values as the peaks for clusters with $\omega > 15°$. However, the relative amplitudes of the peaks differ in the sense that the percentage of clusters that are metal rich ([Fe/H] > –0.8) is about 43% in the $\omega < 15°$ sample, whereas it is only 16% in the $\omega > 15°$ sample. Liller 1 seems to be the most metal rich globular cluster known, with [Fe/H] = +0.25 ± 0.3 (Frogel et al. 1995). For the clusters near the galactic bulge, there is no clear separation between the metal-poor and metal-rich clusters. The metal-rich and metal-poor clusters have velocity dispersions $\sigma_{los} = 77 \pm 14$ and 126 ± 20 km s^{-1}, respectively. There is no evidence, however, for a more rapid rotation of the metal-rich clusters.

4.3 Comparison with stellar populations

In both giant and dwarf elliptical galaxies, there is considerable evidence that their globular clusters do not have the same metallicities, spatial distributions, and kinematics as their stellar populations. In our Galaxy, on the contrary, the four following comparisons suggest that the clusters and the stellar populations away from the galactic bulge region are very similar. First, Armandroff (1989) has shown that the metal-rich disk subsystem has approximately the same [Fe/H], V_{rot}, σ_{los}, and scale height as the so called thick disk stellar population that has been identified in a large number of studies (cf. Gilmore 1989 for a review). Second, studies of the number densities of globular clusters and RR Lyrae variables as a function of the distance to the galactic centre R have shown that they both approximate R^{-n} fall-offs, with n in the range of 3 to 3.5 (Saha 1985, Zinn 1985). There is additional evidence that the clusters and halo stars have similar distributions. Third, the [Fe/H] distributions of the subdwarfs and globular clusters have very nearly the same mean values (Laird et al. 1988). Fourth, the horizontal-branch morphology of the field is not grossly different from that of globular clusters lying in the same zone of R (Kraft 1989).

While the study of the galactic bulge is still in its infancy, there is little question that at least its composition is different from that of the globular clusters within the same area of the sky. But away from the galactic bulge, there is no strong evidence to suggest that the globular clusters and the halo and thick disk stellar populations have different properties. See Zinn (1996) for a recent review.

4.4 Age spread among galactic globular clusters

A precise knowledge of absolute ages of galactic globular clusters, which would provide a lower limit to the age of the universe and hence an upper limit to

the Hubble constant, is still out of reach, due to uncertainties in theoretical models of stellar evolution and in basic calibrations (e.g., absolute luminosities of subdwarfs). According to standard pictures for the formation of the Galaxy (Eggen, Lynden-Bell, & Sandage 1962), the system of globular clusters formed during the rapid dynamical collapse of the protogalactic cloud, a process which should have lasted no more than 1 Gyr (cf. Sandage 1990 for the exact meaning of "rapid"). Fortunately, the relative ages of the galactic globular clusters are more precisely known than their absolute ages, providing evidence for a spread in age among them.

The two galactic globular clusters NGC 288 and NGC 362 are central to recent claims (Bolte 1989, Green & Norris 1990, VandenBerg et al. 1990, Sarajedini & Demarque 1990) of large age differences ($\sim$ 3 Gyr) between galactic globular clusters. But see Stetson et al. (1996). The claimed age differences are derived from stellar evolution models using assumed CNO abundances, whose uncertainties of about a factor of three could account for an apparent 2-Gyr age difference. Dickens et al. (1991) have accurately measured abundances in red giants in NGC 288 and NGC 362 and find that the Fe abundance and the sum of the C, N, and O abundances are essentially the same in every star studied, thus eliminating composition differences and confirming the reality of the age spread.

There are a few clusters — Pal 12, Ruprecht 106, Arp 2, Terzan 7, and IC 4499 — which seem to be unambiguously younger than most globular clusters. With regard to Pal 12 ([Fe/H] $\simeq$ –1.1) and Ruprecht 106 ([Fe/H] $\simeq$ –1.6), how *young* they are depends on what the metal abundances really are, but an age *difference* of $\sim$ 3 Gyr compared to clusters of similar [M/H] seems required (Bolte 1993). The complexity is further increased by the report by Buonanno et al. (1994) of the existence of "young" metal-poor and metal-rich galactic globular clusters. Arp 2, with [Fe/H] $\simeq$ –1.8, is $\simeq$ 3 Gyr younger than the group of the metal-poor clusters (Buonanno et al. 1995a), while Terzan 7, with [Fe/H] $\simeq$ –0.49, is $\simeq$ 4 Gyr younger than 47 Tucanae, another galactic globular of similar metallicity (Buonanno et al. 1995b). IC 4499 is the most recently studied such young globular (Ferraro et al. 1995b). See also van den Bergh (1993c) and Stetson & West (1994) about NGC 6287, which could be the oldest galactic globular cluster.

Another interesting point comes from the 5 best studied clusters with [Fe/H] $\sim$ –2, viz. M68, M92, NGC 6397, M3, and M13, which show a remarkable similarity in age with all values within 0.3 Gyr (VandenBerg et al. 1990; Bolte 1993).

Richer et al. (1996) use the 36 globular clusters with the most reliable age data. These clusters span galactocentric distances from 4 through 100 kpc and cover a metallicity range from [Fe/H] $\simeq$ –0.6 to –2.3. They find that the majority of the globular clusters form an age distribution with a dispersion $\sigma(t)$ $\simeq$ 1 Gyr, and a total age spread smaller than 4 Gyr. Clusters in the lowest metallicity group ([Fe/H] < –1.8) have the same age to well within 1 Gyr, at all locations in the Galaxy halo, suggesting that star formation began throughout the halo nearly simultaneously in its earliest stages. Richer et al. (1996) find no

statistically significant correlation between mean cluster age and galactocentric distance (no age gradient) from 4 to 100 kpc.

The above facts would favor the scenario of Searle & Zinn (1978), in which galaxies are built from the hierarchical merging of smaller subunits in a formation process characterized by the chaotic nature of the collapse, and occurring over a period of a few billion years, several times longer than in the original Eggen et al. (1962) model (but see Sandage 1990 and §5 below). Depending on the adopted value of the Hubble constant $H_\circ$, there is a potential conflict between the age of the Universe and the age of the globular clusters (see, e.g., Bolte & Hogan 1995).

4.5 Implications for the formation and evolution of our Galaxy and its globular clusters

The properties of the halo cluster system that have the largest impact on the theories of the formation and evolution of the Galaxy and its globular clusters are:

(i) the low mean V_{rot} of the halo;

(ii) the weakness of the [Fe/H] gradient with R;

(iii) the wide range in [Fe/H] at every R;

(iv) the lack of correlation between V_{rot} and [Fe/H];

(v) the range in age of several billion years (Gyr) between clusters of the same [Fe/H];

(vi) the systematic variation in horizontal-branch morphology with R.

Recent investigations (e.g., Deliyannis et al. 1990, Lee et al. 1990, 1994) have cast doubt on the viability of the second-parameter candidates other than age (although challenged by Stetson et al. 1996). See Fusi Pecci et al. (1996) for a review. A case has been made for the importance of stellar density (Buonanno 1993) for the morphology of the horizontal branch: see Fig. 9.8 below from Buonanno et al. (1985a). If age is the second parameter, then point (vi) above indicates that the Galaxy evolved from the inside out (Searle & Zinn 1978). Approximately the inner 8-kpc volume of the halo is then older on average by a few Gyr and much more homogeneous in age than the outer halo. Did the inner halo undergo the kind of rapid collapse envisioned by Eggen et al. (1962, see Sandage 1990), while the outer halo was built over several Gyr by the merger of several dwarf galaxies, as argued by Searle & Zinn (1978)?

The properties of the disk globular clusters that are most important from the point of view of galactic evolution are their ages and metallicity gradients with R and |Z|. These are nearly open questions, however, for very few disk globular clusters have been precisely dated.

4.6 Correlations between various properties of galactic globular clusters

A fundamental problem in globular cluster study lies in the determination of the extent to which their properties are either universal or dependent on characteristics of the parent galaxy. The identification of correlations and trends between various properties of galactic globular clusters provides clues which can be used to test and constrain theoretical models of cluster formation and evolution. Earlier work includes the pioneering study by Brosche (1973), along with the studies by Peterson & King (1976), Brosche & Lentes (1984), Chernoff & Djorgovski (1989), Djorgovski (1991), and Covino & Pasinetti Fracassini (1993), among others. Djorgovski & Meylan (1994) gives the most extensive and up-do-date study of this kind. They use a set of 13 cluster parameters, viz.:

– the absolute visual magnitude, M_V;
– the concentration parameter, $c = \log\,(r_t/r_c)$;
– the log of the core radius in parsec, r_c;
– the log of the half-light radius in parsec, r_h;
– the central surface brightness in the V band, $\mu_V(0)$;
– the average surface brightness in the V band within r_h, $\langle\mu_V\rangle_h$;
– the log of the central luminosity density in $L_{\odot V}/\mathrm{pc}^3$, ρ_0;
– the log of the central relaxation time in years, t_{rc};
– the log of the half-mass relaxation time in years, t_{rh};
– the metallicity, [Fe/H];
– the log of the central velocity dispersion in $\mathrm{km\,s^{-1}}$, σ;
– the log of the distance from the galactic centre in kpc, R_{gc}; and
– the log of the distance from the galactic plane in kpc, Z_{gp}.

The definitions of these quantities, error estimates, and other details can be found in the following data bases: Djorgovski & Meylan (1993b), Peterson (1993a), Pryor & Meylan (1993), Trager et al. (1993), and Djorgovski (1993b). Among the 13 quantities mentioned here, only 9 are measured independently: $\langle\mu_V\rangle_h$ is derived from the M_V and r_h; ρ_0 is derived from the $\mu_V(0)$, c, and r_c; t_{rc} is derived from the M_V, c, and r_c; and t_{rh} is derived from the M_V, and r_h. This may cause spurious correlations.

The first striking thing about the data on globular clusters is the vast range they span in many of their properties, e.g. luminosity and density, more so than either elliptical or dwarf galaxies (cf. Djorgovski 1993a for comparisons). Most core or central parameters span a larger range than the corresponding half-light ($\sim$ half-mass) quantities. Qualitatively, this may be understood as a consequence of dynamical evolution which operates faster at core scales, where the reference (“relaxation”) time scales are shorter, by up to a factor of a hundred. It is worth noting that clusters tend to increase the range of their properties as time proceeds. This “stretching of properties” of clusters is inevitable: even if the distributions of cluster properties started as δ-functions, some spread would occur over a Hubble time, already for no other reason than the different tidal effects from one cluster to the other.

The observed fact that the half-mass relaxation times span over two orders of magnitude, and the central relaxation times over some five orders of magnitude, practically guarantees that the globular cluster population will contain a range of objects at all stages of dynamical evolution. Using cluster ellipticities and orientations from White & Shawl (1987), Djorgovski & Meylan (1994) find that these two quantities do not correlate with any other cluster parameters. They also used the ratios Z_{gp}/R_{gc}, which are a statistical measure of the orbit inclinations, without obtaining any new insights.

Luminosity correlations. Luminosity is perhaps the most fundamental observed quantity characterizing a stellar system. For a set of old stellar systems it is a good relative measure of the baryonic mass. Many other properties correlate with luminosity for elliptical and dwarf galaxies; not so for globular clusters (cf. Djorgovski 1993a for comparisons). The only good correlation with luminosity is that with the velocity dispersion. The only other discernible trends are with the concentration and central surface brightness (or equivalently, central luminosity density). More luminous clusters tend to have higher concentrations and denser cores, but there is a large scatter at every luminosity (see also van den Bergh 1994). Interestingly, neither r_c nor r_h correlates with luminosity; this is in a marked contrast with both elliptical and dwarf galaxies, for which the corresponding correlations are excellent (see, e.g., Kormendy 1985).

It has recently been found (Bellazzini et al. 1996) that the correlation between luminosity and core parameters is stronger for clusters lying outside the solar circle than for those inside, which is consistent with the hypothesis that the correlation is primordial, but has been erased by subsequent evolution where evolution time scales are short enough.

Trends with the position in the Galaxy. Globular clusters live in the tidal field of the Galaxy, and are subject to tidal shocks due to both bulge and disk passages (Ostriker et al. 1972; Chernoff & Shapiro 1987; Aguilar et al. 1988; Chernoff & Weinberg 1990, and references therein). Moreover, properties of newly formed clusters may well depend on their position in the proto-Galaxy (e.g., Fall & Rees 1977, 1985; Murray & Lin 1992). It is thus reasonable to expect that some correlations of cluster properties with R_{gc} and/or Z_{gp} will be found. A generic expectation is that clusters closer to the galactic centre will be more dynamically evolved, as tidal shocks accelerate their internal evolution towards the core collapse or dissolution. Chernoff & Djorgovski (1989) analysed the frequency of occurrence of collapsed clusters as a function of position in the Galaxy, and found them to be highly concentrated towards the galactic centre and plane. This trend continues for non-collapsed clusters, in order of decreasing concentration. Djorgovski & Meylan (1994) confirm and extend their findings by looking at the correlations of core parameters with R_{gc} and Z_{gp}. Clusters at smaller R_{gc} tend to have smaller and denser cores and higher concentrations, and thus also shorter central relaxation times. Similar trends are seen when Z_{gp} is used instead of R_{gc}. Indeed, the theory by Fall & Rees (1985) predicts a radial trend of the mean cluster densities, bound by the scaling laws given by the thermal instability ($\rho_h \sim R^{-1}$) and by tidal truncation ($\rho_h \sim$

R^{-2}, following the density law of the dark halo), although other theoretical explanations are certainly possible (cf. Surdin 1995).

Fig. 4.2. Correlations between the core parameters (from Djorgovski & Meylan 1994, Fig. 10). Clusters with smaller cores have higher concentrations, higher central surface brightness and luminosity densities, and therefore also shorter central relaxation times. This is as expected from a family of objects with a roughly constant initial core mass, evolving towards core collapse. All three principal parameters, c, r_c, and $\mu_V(0)$, are measured independently, and correlations are thus real. The correlation involving t_{rc}, as in the lower left panel, is entirely artificial, by mere definition of t_{rc}.

Correlations of core properties. Some of the best correlations of globular cluster properties are those between the various core parameters and concentrations. They are displayed in Fig. 4.2. All three principal quantities r_c, $\mu_V(0)$, and c are measured independently; correlations among them are real. The correlation between the core radius, r_c, and the central surface brightness, $\mu_V(0)$, has been noted by Kormendy (1985). On the other hand, the spectacular correlation between t_{rc} and r_c is entirely artificial: it reflects the derivation

of t_{rc}, which depends on $r_c^{3/2}$ (close to the apparent slope of the correlation), and other quantities which also correlate with r_c.

Fig. 4.3. Velocity dispersion correlations (from Djorgovski & Meylan 1994, Fig. 12). These are the best non-trivial correlations known for globular clusters. The corresponding scaling laws are indicated in the upper left of each panel, and the Pearson (r) and Spearman rank (s) correlation coefficients are listed in the lower right of each panel. The lower right panel shows a bivariate correlation, where core radius is used as a "second parameter" to improve the corresponding correlation shown in the upper right panel.

Metallicity non-correlations. Unlike elliptical and dwarf galaxies, globular clusters show no correlations between metallicity and luminosity or velocity dispersion. The standard explanation for these correlations in galaxies is self-enrichment in the presence of galactic winds. It is thus natural to conclude that globular clusters are not self-enriched systems. This conclusion is also supported by the extreme internal chemical homogeneity of most globular clusters (e.g., Suntzeff 1993; but see Norris & Da Costa 1995 in the exceptional

case of ω Centauri where stars have $-1.8 < [\mathrm{Fe/H}] < -0.8$). Even a single supernova exploding in a still gaseous proto-globular cluster would deposit $\sim 10^{51}$ erg of kinetic energy, which is comparable to the binding energies of globular clusters today, $E_{bind} \sim 10^{50} - 10^{51}$ erg. Thus, with a possible exception of the most massive systems such as ω Centauri, where some chemical inhomogeneities are seen (Dickens & Bell 1976, François et al. 1988, Mukherjee et al. 1992), a still gaseous proto-cluster could be immediately disrupted. Obviously, the exact outcome would depend on many of the as-yet poorly known details of the physics of globular cluster formation. For self-enrichment of globular clusters see, e.g., Smith (1986, 1987) and Morgan & Lake (1989).

Velocity dispersion correlations. Aside from the correlations of core properties, the best non-trivial correlations of globular cluster properties are between the velocity dispersion and luminosity or surface brightness. They are displayed in Fig. 4.3. The corresponding scaling laws are indicated in the upper left of each panel. The central surface brightness expressed in linear units is I_0, the average surface brightness within r_h is I_h, and the total luminosity is L (all in the V band). Since M_V and r_h are not correlated, the correlation between σ and I_h is not simply a consequence of the $L - \sigma$ relation, although they are obviously related. These correlations probably reflect the formation processes of globular clusters more than their subsequent dynamical evolution, and therein lies their significance. In the case of the galactic globular clusters, the relation between velocity dispersion and luminosity ($L - \sigma$) has been already discussed by Meylan & Mayor (1986), Paturel & Garnier (1992), and Djorgovski (1991, 1993a), and the relation between velocity dispersion and surface brightness ($\sigma - \mu$) by Djorgovski (1993a).

The slope of the $L - \sigma$ relation for globular clusters, viz., $L \sim \sigma^{5/3}$, is significantly different from the Faber-Jackson (1976) relation for ellipticals, or its equivalent for dwarf galaxies, viz., $L \sim \sigma^4$. The slope of the $\sigma - \mu$ relation for globular clusters has the *opposite sign* from the corresponding relation for ellipticals and is significantly tighter. The origin of these correlations is not well understood, but they may well reflect initial conditions of cluster formation, and perhaps even be used to probe the initial density perturbation spectrum on a $\sim 10^6$ $M_\odot$ scale. Core radii and concentrations play a role of a "second parameter" in these correlations.

A Multivariate Data Analysis approach: The manifold of globular clusters. Any globular cluster system suffers numerous evolutionary processes, of which some may be connected in very complex ways (like, e.g., the apparent dependence of internal dynamical evolution towards core collapse on the position of the cluster within the Galaxy). While the simple approach of examining individual monovariate correlations of globular cluster parameters provides a useful first look at the system properties, the complexity of the situation calls for a more sophisticated approach. Dealing with a multidimensional data set, subsets of several observables may be connected in multivariate correlations. Simple, monovariate correlations are only a very special and rare case. A multi-

variate statistical analysis may be used to reveal correlations of a more complex nature (e.g., see Fig. 4.3 lower-right panel; see also Djorgovski & Meylan 1994 and Djorgovski 1959).

The data points occupy a volume in an N-dimensional parameter space, where N is the number of input quantities. If any of the input quantities are derived from the others, the data will occupy a volume of dimension M, where M is the number of *independent* input quantities. $N = 13$ and $M = 9$ in Djorgovski & Meylan (1994). If, in addition, any correlations are present in the data, the dimensionality of the volume occupied by the data points, also called the data manifold, will be reduced further. The effective statistical dimensionality of the data manifold, $D \leq M$, gives the number of independent factors which fully describe the data (see the monograph by Murtagh & Heck 1987).

The global manifold of cluster properties has a large statistical dimensionality ($D > 4$), and can be interpreted as a product of many distinct evolutionary processes shaping the observed properties of globular clusters at the present day. A less daunting and more practical approach is to restrict the analysis to some heuristic subsets of variables, where a significant reduction of dimensionality may be found. Consider only the photometric, structural, and dynamical parameters of clusters, M_V, c, r_c, r_h, $\mu_V(0)$, and σ, available for 56 clusters (Djorgovski & Meylan 1994). The statistical dimensionality of this manifold is clearly $D = 3$. This is exactly what can be expected from a family of objects described by King (1966) models. They require 3 input parameters: a scaling of the core radius, a scaling of the surface brightness, and a shape parameter. The fact that the velocity dispersion participates in the manifold suggests that globular clusters have uniform (M/L) ratios (Djorgovski & Meylan 1994; see also, e.g., Brosche & Lentes 1984, Éigenson & Yatsuk 1986, 1989, Fusi Pecci et al. 1993a, and Djorgovski et al. 1993).

The statistical dimensionality of globular clusters is greater than that of elliptical galaxies, for which most global properties form a statistically two-dimensional manifold. However, field elliptical galaxies could be more heterogeneous with a higher dimensionality – three or four (de Carvalho & Djorgovski 1992). Santiago & Djorgovski (1993) have used multivariate statistical analysis to study the relation between the globular cluster content of early-type galaxies and a number of their observed properties.

The fundamental plane correlations for globular clusters. In the parameter space defined by a radius (core or half-light), a surface brightness (central or averaged within the half-light radius), and the central projected velocity dispersion, globular clusters lie on a two-dimensional surface, a plane if logarithmic quantities are considered (Djorgovski 1995). This is analogous to the fundamental plane of elliptical galaxies (Djorgovski & Davis 1987, Faber et al. 1987, Bender et al. 1992; see also Djorgovski & Santiago 1993, Schaeffer et al. 1993). For the core parameters r_c, σ, and $\mu_V(0)$, Djorgovski (1995) obtains a bivariate least-square solution $r_c = \mathrm{f}(\sigma,\mu_V(0))$ which corresponds to the fol-

lowing scaling law:

$$r_c \ \sim \ \sigma^{1.8\pm0.15} \ I_0^{-1.1\pm0.1}. \tag{4.2}$$

Alternatively, a bivariate least-square solution $\mu_V(0) = \mathrm{f}(\sigma, r_c)$, provides a more stable and better fit through surface brightness (Djorgovski 1995) which corresponds to the following scaling law:

$$r_c \ \sim \ \sigma^{2.2\pm0.15} \ I_0^{-1.1\pm0.1}. \tag{4.3}$$

The average of these two solutions is remarkably close to the scaling law expected from the virial theorem:

$$r_c \ \sim \ \sigma^2 \ I_0^{-1} \ (M/L)^{-1}. \tag{4.4}$$

Thus, Eqs. 4.2 and 4.3 are consistent with globular cluster cores being virialized homologous systems with a constant M/L ratio. The corresponding scaling laws on the half-light scale are different, but are nearly identical to those derived from the fundamental plane of elliptical galaxies.

Consequently, the characteristic radii, surface brightness, and central velocity dispersion for globular clusters form statistically two-dimensional manifolds, both on the core and half-light scales. This fundamental plane of globular cluster properties produces the best correlations known for these stellar systems.

Correlations for globular clusters in M31. Similar correlations, involving σ, M_V, $\mu_V(0)$, and $\langle\mu_V\rangle_h$, have been obtained recently for a sample of 21 globular clusters in our neighboring galaxy M31, the Andromeda galaxy (Djorgovski et al. 1996). These globular clusters follow the same correlations between velocity dispersion and luminosity, central, and average surface brightnesses, as do their galactic counterparts. This suggests a common physical origin for these correlations. They may be produced by the same astrophysical conditions and processes operating at the epoch of globular cluster formation in both galaxies. The very existence of these excellent correlations, and their quantitative form as scaling laws, represent challenges and constraints for theories of globular cluster formation (Djorgovski et al. 1996).

5. Formation of globular clusters

The origin of globular clusters requires a physical explanation in any cosmological picture. Globular cluster formation is intricately linked to galaxy formation and evolution, in a way that is difficult to disentangle given the potential multiplicity of simultaneously operating formation scenarios. Because of their great ages, spatial distributions, kinematics, and metallicities, globular clusters stand out as observable clues of the process of galaxy formation and evolution. The standard picture for galaxies — they formed from the gravitational collapse of primordial density fluctuations — may not be applicable in the case of globular

clusters, since in some low-redshift galaxies they still appear to be at the stage of formation. At variance with our Galaxy, globular clusters in other galaxies are not always roughly coeval. Any model should be able to explain the formation not only of single clusters, but also of systems of globular clusters, consisting possibly of a few successive generations. See the extensive review about "Galaxy Formation and the Hubble Sequence" by Silk & Wyse (1993).

5.1 Luminosity function of a globular cluster system

Analysis of the globular cluster luminosity function is important from the perspective of (i) distance estimates (i.e., the globular cluster luminosity function itself is taken to be a "standard candle") and (ii) galaxy formation models and the dynamical evolution of globular cluster systems (Harris 1991, 1996).

In a given galaxy, the number of globular clusters per unit of magnitude interval, $\phi(m)$, is the luminosity function of the globular cluster system, which can also be described in term of absolute magnitude, $\phi(M)$. The globular cluster system luminosity functions now available for several galaxies show that $\phi(m)$ can be simply and accurately described by a gaussian distribution,

$$\phi(m)\ dm = A\ \exp[-(m-m_0)^2/2\sigma^2]\ dm, \tag{5.1}$$

where A is the simple normalization factor representing the total population N_t, m_0 is the mean or peak (turnover) magnitude of the distribution, and σ is the dispersion.

To first order, globular cluster system luminosity functions in different galaxies can then be compared through the two parameters M_0, the absolute magnitude at the turnover, and σ, the dispersion. Over a broad range of systems (Hubble type), the turnover absolute magnitude M_0 is nearly independent of parent galaxy size and environment. For 138 galactic globular clusters, Abraham & van den Bergh (1995) obtain $\langle M_0 \rangle = -7.41 \pm 0.11$ mag and $\sigma =$ 1.24 mag. An unweighted mean for the M_0 of the galaxies in Table 2 of Harris (1991) yields $\langle M_0 \rangle = -7.13 \pm 0.43$ mag. The intrinsic dispersion σ may be systematically a bit larger for the giant ellipticals (for which the best functional fits are reached consistently at $\sigma \simeq 1.4$) than for the other systems (for which $\sigma \simeq 1.2$ seems preferable). The uniformity in $\langle M_0 \rangle$ is all the more remarkable when we consider that the galaxies studied represent at least three distinguishable different processes of galaxy formation (dwarf ellipticals, giant ellipticals, and the spheroids of disk galaxies) as well as a large range of dynamic erosion mechanisms.

The first order similarity of the globular cluster luminosity function from galaxy to galaxy has become increasingly well justified from a purely observational point of view. The physical processes which might have dictated the formation and evolution of a "universal" globular cluster luminosity function are not yet understood, but several possibilities on theoretical grounds now exist (e.g., Fall & Rees 1988, Harris 1991, Jacoby et al. 1992).

5.2 Specific frequency of a globular cluster system

A useful quantity allowing the intercomparison of the globular cluster populations around different galaxies is defined by Harris & van den Bergh (1981). It is called the "specific frequency S_N" of globular clusters and represents the total number of globular clusters per unit $M_V = -15$ of host galaxy luminosity,

$$S_N \equiv \ N_t \ 10^{0.4(M_v+15)}, \tag{5.2}$$

where N_t is the total number of clusters integrated over the entire globular cluster luminosity function and M_V is the absolute visual magnitude of the galaxy.

Although specific frequencies show very large galaxy-to-galaxy variations, there is a clear tendency for S_N to increase along the sequence from late-type to early-type galaxies. Characteristic values of S_N range from $S_N \lesssim 1$ for spiral and irregular, to $S_N \simeq$ 2-3 for normal elliptical galaxies in low-density environments, to $S_N \simeq$ 5-6 for ellipticals located in rich clusters; cD galaxies located at the centres of rich clusters have the largest known specific globular cluster frequencies, typically $S_N \simeq$ 10-20. The prototype of these high-S_N galaxies is NGC 4486 $\equiv$ M87, the central cD in the Virgo cluster, which has a population of at least 15,000 globular clusters, with $S_N = 14$. Our Galaxy has $S_N = 0.5$. The fact that the S_N values of elliptical galaxies in rich clusters are systematically higher than those of their counterparts in low-density regions, suggests that the local galactic environment plays a key role, along with the galaxy type, in determining globular cluster frequencies. See Harris (1991).

Several scenarios have been proposed to explain the origin of the observed systematic variations in globular cluster specific frequency (see van den Bergh 1993d, 1995b, and Hesser 1993 for interesting discussions):

(1) Harris (1981) and van den Bergh (1982) suggest that elliptical galaxies might have been, for some unspecified reason, more efficient than spiral galaxies at forming globular clusters, and that the higher S_N values for ellipticals in rich clusters compared to those in the field may be accounted for by assuming that the latter had experienced a greater number of past mergers with low-S_N spiral galaxies.

(2) Fabian et al. (1984) and Fall & Rees (1985) suggest that globular clusters might form from gas which condenses out of cooling flows in the dense cores of rich galaxy clusters. This accounts for the high S_N values of some cD galaxies in galaxy cluster cores, but fails to explain why some cD galaxies which appear to sit in the middle of large cooling flows have "normal" S_N values while other high-S_N galaxies reside in rich clusters which have no cooling flows at present.

(3) Considering that central cluster galaxies, cD galaxies in particular, are thought to have grown by cannibalism and/or mergers, and since normal cluster galaxies have much lower S_N values than central galaxies, van den Bergh (1984, and references therein) argues that such mergers will reduce the

S_N values of the central galaxies. The specific frequencies of central cluster galaxies must therefore have been even greater originally than they are at present. Van den Bergh suggests that "central galaxies in rich clusters were special *ab initio*".

(4) Muzzio (1987, and references therein) explored the possibility that globular clusters might be stripped from the outer halos of galaxies in the dense environments of rich clusters, and captured later on by massive galaxies residing at the bottom of the galaxy cluster potential well. However, their N-body simulations indicate that the magnitude of this effect is rather small. See also West et al. (1995).

(5) There is increasing observational evidence (e.g., Schweizer 1987, Holtzman et al. 1992, 1996) which supports the hypothesis that globular clusters form during the interactions or mergers of galaxies (Ashman & Zepf 1992, Zepf & Ashman 1993, and references therein). Elliptical galaxies presumably underwent more frequent merging than spiral galaxies, and accordingly would be expected to have more abundant globular cluster systems.

(6) Zinnecker et al. (1988) and Freeman (1990, 1993) suggest that many of the globular clusters seen around high-S_N galaxies may actually be the surviving cores of nucleated dwarf elliptical galaxies. Tentative support for this view comes from observational similarities — such as luminosities, integrated colors, and velocity dispersions — between nucleated cores and globular clusters. See the numerical simulations by Bassino et al. (1994).

Interestingly enough, there is plethora of scenarios for forming globular cluster systems; nevertheless, none of them is able to explain without major ad hoc tuning why the faintest galaxy known to have globular clusters has, by far, the highest known specific frequency value, viz. $S_N = 73$. It is the Fornax dwarf spheroidal galaxy (dSph), which has five globular clusters for an absolute magnitude $M_V = -12.3$ (Harris 1991). Is the Fornax dwarf spheroidal galaxy, located at about 130 kpc from the centre of our galaxy, a genuine and unique exception? Is such a unique faint low-density galaxy, which would be hardly detectable around M31, located by chance in our galactic neighborhood? See Minniti et al. (1996) for globular clusters around dwarf elliptical galaxies.

5.3 Globular cluster formation models

A fully consistent model for globular cluster formation has proven to be a formidable theoretical challenge and is still missing. Nevertheless, some scenarios/models have been developed during the last few decades, and represent steps towards a general understanding of the way globular clusters (and galaxies) form. The studies can be sorted into two broad families: (i) globular clusters were the first condensed systems to form in the early universe (Peebles & Dicke 1968) or during conditions which existed only in protogalactic epochs (Fall & Rees 1985), (ii) globular clusters originated in larger star-forming systems that later merged to form the present galaxies (Larson 1993, 1996).

The conventional picture (von Weizsäcker 1955) describes the globular clusters as formed along with the halo field-population stars during the initial collapse of the protogalaxy, a collapse described as rapid, smooth, and homogeneous in Eggen, Lynden-Bell, & Sandage (1962; but see Sandage 1990).

This view was challenged by Peebles & Dicke (1968) who consider the expected properties of the first bound systems to have formed out of the expanding universe. They point out that the coincidence between, (i) the properties of globular clusters, and (ii) the computed mass and estimated radius of objects at the time of first fragmentation into stars, argues strongly for the general validity of the view that these first systems are protoglobular clusters. The predicted masses and radii, of order $10^6\ M_\odot$ and 10 pc, are typical of the observed values for globular clusters. Consequently, globular clusters would reflect the Jeans mass at recombination, when the temperature dropped below 10^4 K and the mean density of the universe was about 10^9 times higher than at present. In other words, the smallest gravitationally unstable clouds produced from isothermal perturbations just after recombination could be identified as the progenitors of globular clusters.

An interesting refinement of the above scenario is the possibility that globular cluster formation might have been "biased" in the sense that only those $\sim 10^6 M_\odot$ peaks in the fluctuating density field that exceeded some critical global threshold were able to form globular clusters. Some aspects of biased globular cluster formation are presented in Peebles (1984), Couchman & Rees (1986), and Rosenblatt et al. (1988), although all these studies focus on the specific case of a universe dominated by cold dark matter. West (1993) presents a simple model of biased globular cluster formation which relates the efficiency of globular cluster formation to both galaxy type and local environment. While the magnitude of this effect is clearly sensitive to assumptions about biasing parameters which are poorly constrained, this study shows that, for quite reasonable assumptions about the biasing process, it is possible to reproduce the observed variations in globular cluster populations remarkably well. Biased formation may also explain why other globular cluster properties, such as the luminosity function, appear to be universal.

Fall & Rees (1985) argue that globular clusters would form in the collapsing gas of a protogalaxy. The Jeans mass M_J of a spherical cloud with a temperature T_c confined by an external pressure p_e is roughly

$$M_J \approx (kT_c/m_H)^2 G^{-3/2} p_e^{-1/2}. \tag{5.3}$$

A natural value for T_c is 10^4 K, where the radiative cooling rate drops precipitously, and a natural value for p_e is $\rho_g v_g^2$, where ρ_g is the mean density within a protogalaxy, and v_g is a typical collapse or virial velocity: the result is then $M_J \sim 10^6 M_\odot$. Their starting point is the generally accepted view that fragmentation and star formation can only occur when the gas is able to cool in a free-fall time (Rees & Ostriker 1977, Silk 1977). Under these conditions, any gas at the virial temperature, of order 10^6 K, will be thermally unstable and will develop a two-phase structure. Fall & Rees suggest that the condensation of cold clouds progressively depletes the hot gas in such a way that its cooling

and free-fall times remain comparable. The clouds, which have temperatures near 10^4 K and densities several hundred times that of the surrounding hot gas, are gravitationally unstable if their masses are of order $10^6 M_\odot$. They identify these objects as the progenitors of globular clusters and speculate on their later evolution. Such events must occur at redshifts less than 10 because a thermal instability is not effective when Compton scattering by cosmic background radiation is the dominant cooling process. Some aspects of the work of Fall & Rees (1985) complement the suggestion by Gunn (1980) and McCrea (1982) that globular clusters formed in the compressed gas behind strong shocks. In contrast with these previous discussions, Fall & Rees emphasize that the clouds must cool slowly at temperatures just below 10^4 K to imprint a characteristic mass of order $10^6 M_\odot$. They also show that the heating of much smaller clouds by X-rays from the hot gas would inhibit the formation of field stars and small clusters during the initial collapse. In a follow-up study, Kang et al. (1990) examine in more detail the thermal history of metal-free gas overtaken by radiative shocks with velocities characteristic of gravitationally induced motions inside a typical protogalaxy. See also Ashman (1990) and Murray & Lin (1991, 1992).

As an alternative view to the Eggen et al. (1962) galaxy formation scenario, Searle & Zinn (1978) consider protogalaxies as very lumpy systems. Galaxies are built from the hierarchical merging of smaller subunits. As a result of the more chaotic nature of the collapse in the Searle & Zinn (1978) scenario, the formation process occurs over a period of a few billion years, several times longer than in the original Eggen et al. (1962) model. Within the framework of the Searle & Zinn (1978) model, it is plausible that the collision and coalescence of the subunits could lead to conditions appropriate for the Fall & Rees (1985) model, as discussed by Kang et al. (1990). However, an emphasis on the lumpy nature of protogalaxies promotes the consideration of other ideas. Larson (1986) notes that in observed star-forming regions, the fraction of the parent cloud that ends up in stars is small. In complexes like Orion only about 10^{-3} of the original cloud will end up in a bound star cluster. The inference is that the progenitors of globular clusters must have had masses in excess of 10^8 $M_\odot$. Such objects may be identified with the protogalaxy lumps of Searle & Zinn (1978).

Globular clusters may form during the interaction or merger of galaxies, complicating further the previous scenarios. Ashman & Zepf (1992) and Zepf & Ashman (1993) suggest that galaxies in which globular cluster formation is currently occurring are systems which are interacting with larger galaxies (e.g., the Large Magellanic Cloud interacting with the Galaxy, see §5.6 below). They also describe indirect evidence that some of the globular clusters in massive galaxies were formed as the result of interaction or merger of pre-existing disk galaxies, but such scenarios have difficulties in explaining the sheer numbers of clusters in elliptical and dwarf elliptical galaxies.

Some of the above studies are based, among other simplifications, on the hypothesis that the masses of globular clusters are confined to a remarkably narrow range, roughly 10^5-10^6 $M_\odot$. This may be plausible for the rich glo-

bular clusters in the inner part of the Galaxy, but does not account easily for the properties of the globular clusters at large galactocentric distances. In our Galaxy, the present (possibly dynamically evolved) masses of globular clusters span a range of more than three orders of magnitude (see §2). In recent years, the mass function of globular cluster systems, which has an extremely similar shape in all galaxies (Harris & Pudritz 1994, McLaughlin & Pudritz 1996), has emerged as a major clue to the formation processes. The number of clusters per unit mass is nearly constant for masses less than $\simeq 10^5 M_\odot$, a limit corresponding to the turnover point in the luminosity function (Harris 1991). For clusters with higher masses, a simple power-law form $N(M) \propto M^{-\gamma}$ applies extremely well, with exponent $\gamma \simeq 1.7$ - 1.9 for the spirals and dwarf ellipticals, and $\gamma \simeq 1.6$ for the giant ellipticals (Surdin 1979, Harris & Pudritz 1994, McLaughlin & Pudritz 1996). Cluster dynamical evolution (stellar mass loss and tidal shocking) must influence the low-mass end of the distribution, but the ubiquity of the breakpoint at $\simeq 10^5 M_\odot$ and the similar slope at higher masses, both across a large diversity of environments, suggest that the major part of the mass function of globular cluster systems must be a characteristic of the formation process. These mass-function slopes $\gamma \simeq 1.5$ - 2.0 are also similar to the values found for the mass functions of both the open clusters and the dense molecular clumps in which they are born. Harris (1996) suggests that these observational constraints rule out the above theories in which globular clusters, assumed to form by thermal instability, have pregalactic origin or arose in environmental conditions present only in protogalactic epochs (Peebles & Dicke 1968, Fall & Rees 1985, Rosenblatt et al. 1988, Murray & Lin 1990b, 1992, Vietri & Pesce 1995).

The alternative is that there is nothing special about globular cluster formation: it represents only the high-mass tail of the general process of star cluster formation which is happening nowadays in any galaxy which contains a decent supply of cool gas (Larson 1990, 1993, 1996, Harris & Pudritz 1994, Patel & Pudritz 1994).

5.4 Collapse, fragmentation, and initial mass function

A major goal of studies of globular cluster formation is to understand, through fragmentation, the spectrum of masses with which stars form, since the initial mass spectrum plays a fundamental role in determining the observed properties of stellar systems and their subsequent dynamical evolution. We briefly mention hereafter a few recent studies related to fragmentation of clouds into stars.

The fragmentation process of molecular clouds has been investigated by Chièze (1987), taking into account the observed relations $M \propto R^2$ and $\sigma \propto R^{1/2}$, between the mass M, the radius R and the internal velocity dispersion σ of molecular clouds, relations first noticed by Larson (1981). Chièze (1987) shows that interstellar molecular clouds which are close to gravitational

instability exhibit precisely the same scaling laws, provided they interact with a constant pressure environment. He suggests that these conditions may trigger the fragmentation of clouds. See also Chièze & Pineau des Forêts (1987) for fragmentation of low-mass molecular clouds, de Boisanger & Chièze (1991) for formation of molecular clumps in an inhomogeneous radiation field, and Renard & Chièze (1993) for the behavior of critical Jeans mass close to thermal instability. Murray & Lin (1989a,b) have studied proto-globular cluster fragmentation in the case of thermal and gravitational instabilities, respectively. See also Di Fazio (1986) in the case of gravitational instabilities. Myers & Fuller (1993) find clear relations, in the form of simple power laws, between the line width of a dense core observed in the 1.3 cm lines of NH_3 and the luminosity and mass of the most massive stars associated with this core. From their study of gravitational formation times and IMF, they predict infall times equal to 1-2, 4-8, and 1-12 $\times$ 10^5 yr for stars of mass 0.3, 3, and 30 $M_\odot$, respectively.

Dynamical mixing in molecular clouds in relation to the origin of metal homogeneities in globular clusters have been investigated by Chièze & Pineau des Forêts (1989) and Murray & Lin (1990a).

The difficulty of predicting the initial mass function (IMF) comes from the fact that a large number of different physical processes are likely to take place during star formation, including cloud fragmentation, fragment coalescence, mass accretion in a disk, stellar wind mass loss, among others. How these processes combine to determine a final stellar mass at a particular time in a cloud, or to determine an average mass spectrum in a composite of clouds, is difficult to simulate in any detail. See Shu et al. (1987) and Adams & Fatuzzo (1996).

In one of the first attempts to determine the IMF, Elmegreen (1985) uses a statistical approach which specifies from physics the mass distribution of stars in a cloud, but not the mass of an individual star. The mass of each individual star is, in such a theory, the result of a large number of independent events, all of which involve combinations of randomly chosen parameters. A combination of fragmentation and accretion processes in hierarchical groupings of forming stars may play an important role in the formation of massive stars (Larson 1982, 1992). As reviewed by Scalo (1986), there is considerable evidence that molecular clouds have complex hierarchical structures and are typically filamentary in shape. A fractal description of star-forming clouds was first propose by Henriksen (1986) and then further explored by Dickman et al. (1990) and Falgarone et al. (1991).

Prescribed IMF of stars reaching the main sequence have been used by Fletcher & Stahler (1994a,b) in order to compute the history of the luminosity function of young clusters still forming within a molecular cloud. In these models, the number of protostars rises quickly but levels off to a nearly constant value which lasts until the dispersal of the cloud.

From a theoretical point of view, understanding the origin of the IMF remains a difficult task, with the result that model predictions are still in their infancy. The same is true from an observational point of view: e.g., although there are some theoretical arguments predicting that low-mass star formation

may be suppressed in regions of high-mass star formation, the observational constraints remain inconclusive. Zinnecker (1996) shows that at least in the case of NGC 3603 there is evidence, from adaptive optics data in H and K bands, that subsolar-mass stars are present. In the case of R136, the core of the 30 Doradus nebula, the IMF could not be probed below about 2 $M_\odot$, but no cutoff has been observed down to this detection limit (Zinnecker 1996).

5.5 Early stellar evolution and violent relaxation phase

From the point of view of dynamics, the most important consequence of the early evolution of the stars is the accompanying loss of mass, which tends to unbind the cluster. Usually, this is modelled as a sudden loss of mass by each star at the end of its main sequence evolution, and usually it is assumed that the mass is ejected instantaneously out of the cluster. Usually the process is handled in terms of a lookup table which provides, for a main sequence star of a given mass, the time and amount of mass loss. A commonly used prescription is that adopted by Chernoff & Weinberg (1990), which was based on work of Iben & Renzini (1983) and Miller & Scalo (1979).

After some early, but still interesting and relevant, investigations by Angeletti & Giannone (1977c, 1980), Applegate (1986) was among the first to revive interest in the dynamical effects of mass loss at the end of main sequence evolution. He used a simple model of relaxation to show how sufficient loss of mass (and the resulting expansion of the cluster) either delayed the onset of relaxation processes or exposed the cluster to the danger of disruption by tidal shocking. More quantitative detail was added to this picture by Chernoff & Weinberg (1990), who did a more careful job of modelling relaxation (by using a Fokker-Planck code), but included only a steady tide. Their results were qualitatively similar, and showed that the mass loss by stellar evolution would always disrupt a cluster with a relatively flat mass function (i.e. $dN \propto m^{-\alpha}dm$ with $\alpha = 1.5$ over the range $0.4 < m < 15\ M_\odot$). Clusters with steeper mass functions would survive without disruption provided that the initial concentration was high enough; they used King models as initial models and found that a model with initial scaled central potential $W_0 = 7$ would survive for $\alpha \geq 2.5$. These results are clearly dependent on the assumed range of the mass function, and somewhat more generalised results will be found in Weinberg (1993a) and Chernoff (1993), where special consideration is given to clusters that disrupt so quickly that relaxation effects can be ignored.

More careful (N-body) modelling of this same problem has been carried out by Fukushige & Heggie (1995). They confirm the qualitative results of Chernoff & Weinberg (1990), but find that the destruction times were underestimated by factors as large as 10 in some cases. The problem appears to arise from the fact that the time scale on which mass is lost by the cluster is not long enough compared to the crossing time, and this leads to failure of the assumption (on which earlier workers depended) that the cluster evolves

through a sequence of quasi-equilibrium models.

These models are based on instantaneous mass loss by each star (at the end of the main sequence lifetime appropriate to its initial mass), and are of importance for a theoretical understanding of the evolution of globular clusters. They may, however, be oversimplified, especially in the first intensive phase of mass loss. During this early phase (roughly the first 10^7yr), massive clusters may well have possessed substantial quantities of unejected gas, while those of lower mass may have generated an outflow in the form of a cluster wind (Smith 1996). Such a wind can have the effect of expelling the residual gas left from the star formation process itself. The dynamical consequences of this expulsion have been considered mostly in the context of open star clusters (e.g. Tutukov 1978, Hills 1980, Mathieu 1986 and references therein, Lada & Lada 1991 and references therein). In the context of globulars, some N-body modelling of these problems has been carried out recently by Goodwin (1996).

Large amounts of irregular mass loss may induce violent changes of the gravitational field of a newly formed globular cluster. This phase of dynamical mixing changes the statistics of stellar orbits on a time scale of the order of the crossing time ($\sim 10^6$ yr), consequently, this is an encounterless relaxation phenomenon. It has been named violent relaxation and was first addressed by Lynden-Bell (1962, 1967), Hénon (1964), and King (1966). The violent relaxation leads quickly to the smooth light distribution typical of King-Michie clusters, which are characterized by a steady dynamical evolution with relaxation due to stellar encounters, leading slowly, after a few Gyr, to core collapse and/or evaporation.

More recent theoretical discussions about the fundamentals of violent relaxation are found in Shu (1978, 1987), Tremaine et al. (1986), Kandrup (1987), Tanekusa (1987), Aarseth et al. (1988), Funato et al. (1992), and Spergel & Hernquist (1992).

It is known that, in large systems like globular clusters, primordial binaries are left largely intact by early phases of violent relaxation (Vesperini & Chernoff 1996). Their importance for subsequent evolutionary stages is one of the main themes of §9.5.

5.6 Formation of globular clusters in the Magellanic Clouds

The spread in age among the Magellanic Clouds clusters is much larger than for those in the Galaxy, but a recent study of Hodge 11, a globular cluster in the Large Magellanic Cloud, points towards an age identical to that of the galactic globular M92 (Mighell et al. 1996). Consequently, the oldest star clusters in the Large Magellanic Cloud and in the Galaxy appear to have the same age. From the histogram of the ages of the star clusters in the Large Magellanic Cloud (LMC), it is as conspicuous as it is surprising to see that about one half of all clusters are younger than $\sim 10^8$ yr (van den Bergh 1981, Elson & Fall 1988). It looks as if the LMC managed to produce in the last $\sim 10^8$ yr as many

clusters as during the last $\sim 10^{10}$ yr. Are we lucky enough to witness a burst of formation of star clusters? Probably not. Most of the currently forming star clusters are not massive enough to remain in a magnitude limited catalog for more than $\sim 10^8$ yr. Dynamical disruption as well as fading away bring their integrated luminosity below the threshold of present catalogs.

But there is little doubt that rich star clusters, which have no equivalent in our Galaxy, are currently forming in the Magellanic Clouds. However, there has been some debate about two points:

First, it is not sure that the very rich and young LMC clusters are truly the progenitors of their older globular counterparts. The masses of galactic globular clusters span quite a broad range, from less than 10^4 $M_\odot$ to over 10^6 $M_\odot$. Many of the young LMC clusters fall comfortably in this range: e.g., M_{tot}(NGC 1850) = 6 $\times$ 10^4 $M_\odot$ (Fischer et al. 1993a), M_{tot}(NGC 1866) = 6 $\times$ 10^5 $M_\odot$ (Lupton et al. 1989), M_{tot}(NGC 1866) = 1 $\times$ 10^5 $M_\odot$ (Fischer et al. 1992a), M_{tot}(NGC 1978) = 9 $\times$ 10^5 $M_\odot$ (Meylan et al. 1991c), M_{tot}(NGC 1978) = 2 $\times$ 10^5 $M_\odot$ (Fischer et al. 1992b), M_{tot}(NGC 2164) = 2 $\times$ 10^5 $M_\odot$ (Lupton et al. 1989), and M_{tot}(NGC 2214) = 4 $\times$ 10^5 $M_\odot$ (Lupton et al. 1989). The differences between the estimates concerning one given cluster, such as NGC 1978, are model dependent and come also from the fact that radial velocities of individual stellar members of Magellanic clusters suffer from crowding problems, leading to underestimation of the true velocity dispersion. A greater concern resides in the fact that the young objects in the LMC are more closely analogous to the open clusters of the Milky Way, as suggested by similarities in the cluster luminosity functions (Elson et al. 1987a, van den Bergh 1993d, 1995a,b). However, the conspicuous difference between young Magellanic and open galactic cluster systems is in the presence of young massive clusters in the LMC, such as NGC 1866 (young $\sim 10^8$ yr, rich $\sim 10^5$ $M_\odot$, and luminous $\sim 10^6$ $L_\odot$), which has no known counterpart in the disk of the Milky Way. This cluster looks like a genuine globular cluster which may be similar to NGC 1835 in 10 Gyr. The LMC seems able to make one genuine globular cluster in $\sim 10^8$ yr. Considering in the LMC the number of young poor clusters to be about 100, there is a cluster formation efficiency of about 1 poor cluster per 1 Myr, and 1 rich cluster per 100 Myr. The time scale to produce an even more massive cluster (a few times 10^5 $M_\odot$) is longer. It may be very well that there is no special epoch in the LMC history, and the age histogram of the star clusters (of all richness) looks similar over the Gyr. There is just a continuous replenishment of new small clusters as those already formed fade away, and only rarely a fairly massive cluster is formed which manages to remain brighter than the threshold for many years (Renzini 1991). See Fujimoto & Noguchi (1990) for an interesting investigation of dynamical conditions for globular cluster formation, in the specific case of the Magellanic Clouds, by studying hydrodynamical collisions between gas clouds and their subsequent coalescence.

Current observations are consistent with the idea that both the galactic disk and the LMC are currently forming star clusters, but only the LMC

contains young clusters with masses characteristic of globulars (see also Renzini 1991). Dynamical simulations including the combined effects of relaxation, and tidal and binary heatings are consistent with suggestions that the shape of cluster luminosity functions results from evaporation and disruption of low mass clusters (Chernoff & Weinberg 1989, Murali & Weinberg 1996). Since the less massive Magellanic clusters are more susceptible to disruption by various evolutionary processes, the LMC cluster luminosity function will evolve so that it will more closely resemble the luminosity function of the galactic globulars, whose luminosity function has largely been shaped by dynamical selection (Murali & Weinberg 1996).

Second, it has been thought that the old LMC clusters are significantly less massive than their galactic globular cluster counterparts. Meylan (1988b), Dubath et al. (1990) show that, in the case of NGC 1835, a projected velocity dispersion σ_p(core) = 10.1 $\pm$ 0.2 km s^{-1} provides a total mass M_{tot} = 1.0 $\pm$ 0.3 $\times$ 10^6 $M_\odot$, corresponding to a global mass-to-light ratio $M/L_V = 3.4 \pm 1.0$ $(M/L_V)_\odot$. This study shows that when the same kind of dynamical models (King-Michie) constrained by the same kind of observations (surface brightness profile and central value of the projected velocity dispersion) are applied to an old rich Magellanic globular cluster, viz., NGC 1835, the results are similar to those obtained in the case of galactic globular clusters. Consequently, the rich old globular clusters in the Magellanic clouds could be quite similar (in mass and M/L_V) to the rich globular clusters in the Galaxy.

Is the 30 Doradus Nebula a globular cluster progenitor? If a genuine globular cluster were forming right now in the Local Group, there would be probably only one place where this could be happening: within the 30 Doradus Nebula. The LMC star cluster NGC 2070 is embedded in the 30 Doradus nebula, the largest HII region in the Local Group (see Meylan 1993 for a review). The physical size of NGC 2070, with a diameter $\sim$ 40 pc, is typical of old galactic and Magellanic globular clusters. The size of NGC 2070 is also comparable to the size of its nearest neighbor, the young globular cluster NGC 2100, which lies about 53$'$ southeast of 30 Dor. With an age of $\sim 4 \times 10^6$ yr (Meylan 1993, Brandl et al. 1996), NGC 2070 appears slightly younger than NGC 2100 which has an age of $\sim$ 12-16 $\times$ 10^6 yr (Sagar & Richtler 1991). Their masses are also quite similar. For the 30 Dor cluster, Churchwell (1975) estimates the mass of ionized gas larger than 3×10^5 $M_\odot$ and the total mass contained in the stellar cluster larger than 4×10^5 $M_\odot$; extrapolations of the IMF exponent obtained for the high mass stars give total masses from 3×10^4 to 6×10^5 $M_\odot$ within 100$''$ (Meylan 1993); Malumuth & Heap (1994) obtain, from the Hubble Space Telescope (HST) data, a lower limit to the mass within 17.5$''$ equal to 2 $\times$ 10^4 $M_\odot$, while Brandl et al. (1996), from data obtained with the ESO adaptive optics system COME ON+, estimate, from the total K magnitude, the mass within 20$''$ equal to 3×10^4 $M_\odot$, with an upper limit on this value equal to 1.5×10^5 $M_\odot$. A star cluster with a mass of this range and a typical velocity

dispersion of $\sim$ 5 km s^{-1} would be gravitationally bound, a conclusion not immediately applicable to NGC 2070 because of the important mass loss due to stellar evolution experienced by a large number of its stars (see Kennicutt & Chu 1988 and below). For NGC 2100, Lee (1991) finds a total mass M_{tot} = 5 $\times$ 10^5 $M_\odot$. All mass determinations for these two very young Magellanic clusters provide results typical of masses of old galactic globular clusters.

Mass segregation may have been observed in NGC 2070. Brandl et al. (1996) determine, for stars more massive than 12 $M_\odot$, a mean mass-function slope x = 1.6 [x(Salpeter) = 1.35], but this value increases from x = 1.3 in the inner 0.4 pc to x = 2.2 outside 0.8 pc. The fraction of massive stars is higher in the centre of R136, the core of NGC 2070. This may be due to a spatially variable initial mass function, a delayed star formation in the core, or the result of dynamical processes that segregated an initially uniform stellar mass distribution.

In their study of the formation and evolution of rich star clusters, Kennicutt & Chu (1988) use a simple cluster evolution and photoionization model and show that for a cluster like NGC 1866, its initial ionizing luminosity is consistent with the actual ionization requirement of the 30 Dor Nebula. Furthermore, in their later study of the kinematic structure of this object, Chu & Kennicutt (1994) reach the conclusion that 30 Dor and its vicinity will evolve into a supergiant shell as seen in nearby galaxies (see also Hunter et al. 1995).

5.7 Formation of globular clusters in other nearby galaxies

Kennicutt & Chu (1988) have reviewed the question of the formation of young globular clusters and their possible association with giant HII complexes in nearby galaxies. They define a young globular as any object with $B - V <$ 0.5 and a mass exceeding 10^4 $M_\odot$. For the two massive spiral galaxies in the Local Group, the number of young globulars is negligible (zero in the Milky Way; a few marginal candidates in M31). The LMC has a large number of young globulars, whereas the young objects in the SMC are close to the adopted luminosity threshold. Other galaxies in the Local Group seem to contain young clusters.

Outside of the local group, even more massive clusters are apparently forming in starburst galaxies, especially in interacting and merging systems. High angular resolution observations of several merging galaxies have been obtained with the HST. Holtzman et al. (1992) discovered a population of about 60 bright blue pointlike sources concentrated within 5 kpc from the nucleus of NGC 1275, a galaxy thought to be the result of a recent merger. The brightest object has an absolute magnitude $M_V \sim$ –16, with typical M_V from –12 to –14. Ages are of the order of a few 100 $\times$ 10^6 yr or less, with masses between 10^5 and 10^8 $M_\odot$. Subsequent spectroscopic data obtain by Zepf et al. (1995) for the brightest of these sources give an age of about 0.5 Gyr. Whitmore et al. (1993) observed in NGC 7252, another merger remnant, a concentrated population of

40 bright blue pointlike sources with mean $M_V \sim -13$ and mean age of about 100×10^6 yr. O'Connell et al. (1994) observed three such bright clusters in NGC 1569 and NGC 1705; they have M_V between ~ -13.3 and -14.1 and ages larger then 15×10^6 yr. In NGC 4038/4039 (the Antennae), the prototypical example of a pair of colliding galaxies, Whitmore & Schweitzer (1995) observed a population of 700 bright blue pointlike sources. The brightest objects have absolute magnitudes $M_V \sim -15$, and the mean value is $M_V = -11$. The brightest bluest clusters have ages less than 10×10^6 yr. In M82, a starburst galaxy, probably as a consequence of tidal interactions with its neighbor, O'Connell et al. (1995) found over 100 bright blue pointlike sources, with mean $M_V = -11.6$. See Holtzman et al. (1996) for star clusters in interacting and cooling flow galaxies and Forbes et al. (1996) for star clusters in the central regions of kinematically distinct core ellipticals.

In relation to globular clusters formation theory, it is worth mentioning that these extremely luminous young stellar aggregates found in all these interacting/merging galaxies have sizes and estimated masses which overlap with those of the globular clusters. Their luminosity functions have a power-law form similar to those of the open clusters and the more massive globular clusters (see §5.3 above). Systematic spectroscopy of these objects would help estimating the fraction of these bright blue sources which may evolve into genuine old globular clusters. Present spare spectroscopy data strongly support the notion that they are young globular clusters formed during interactions or mergers (Schweitzer & Seitzer 1993, Zepf et al. 1995).

6. Observations providing dynamical constraints

Most dynamical models can be constrained by the same kind of observations, viz. the surface brightness profile and the velocity dispersion profile. These profiles can be constructed from the following observational data: (i) density profiles from star counts, (ii) density profiles from surface brightness measurements, (iii) velocity dispersion profiles from proper motions, (iv) velocity dispersion profiles from stellar radial velocities, and (v) core velocity dispersions from integrated-light spectra. It is worth mentioning at this stage that two models intrinsically very different may have very similar surface density profiles (see §7.7). Another important input parameter is the mass function, which can be reliably obtained from observations only for the upper part of the main sequence, although the HST with WFPC2 provides significant improvement in the sampling of the luminosity function down to stars of about 0.1 $M_\odot$ (see, e.g., Richer et al. 1995 and King et al. 1995).

6.1 Star counts and surface brightness profiles from photographic plates, photomultipliers, and CCD images

Star counts from photographic plates. Around the end of the $19^{\rm th}$ century, the advance of photographic techniques applied to astronomy gave astronomers the opportunity to shift from descriptive work to more quantitative, scientifically more objective studies. It is from a photographic plate of ω Centauri, taken at Arequipa (Peru), with an exposure time of two hours, that Bailey (1893) made what probably was the first extensive star count study of a galactic globular cluster. These data were used by Pickering (1897) in the first important comparisons between observed and empirically guessed theoretical profiles. During the following decades, till the development of CCDs, star counts from photographic plates were intensively used in order to study the distribution of stars in clusters. All these data represent very heterogeneous material which is scattered in the literature and not easily accessible.

Refinements in the theoretical understanding of cluster dynamics led to a strong need for extensive and homogeneous star count data. King (1966) provided, for the first time, a grid of models with different concentrations $c = \log{(r_t/r_c)}$ that approximately incorporated the three most important elements governing globular cluster structure: dynamical equilibrium, two-body relaxation, and tidal truncation (r_t and r_c are the tidal and core radii, respectively; see §7.5 below). These models, being spherical, isotropic, and composed of stars with a single mass, were the simplest that might acceptably represent the star count data. King et al. (1968) demonstrated the success of these models when they published an enormous amount of observational data, viz., star counts for 54 galactic globular clusters.

No similar effort, in bringing a coherent and large data base for star counts in globular clusters, has enlarged and improved the earlier work by King et al. (1968), until the recent publication by Grillmair et al. (1995a). They obtained deep two-color photographic photometry in order to examine the outer structure of 12 galactic globular clusters, using star count analyses. They find that most of their sample clusters show, in their surface density profiles, extra-tidal wings whose profiles have forms consistent with recent numerical studies of tidal stripping of globular clusters (Grillmair et al. 1995b). Two-dimensional surface density maps are consistent, for several clusters, with the expected appearance of tidal tails, with the allowance for the effects of orbit shape, orbital phase, and orientation of our line of sight. The extra-tidal material is identified with stars still in the process of being removed from the clusters, limiting the accuracy of the determination of the tidal radius. Grillmair et al. (1995a) conclude that the stars found beyond the best-fit values of r_t are probably unbound as a result of previous and ongoing stripping episodes. They speculate that globular clusters in general have no observable limiting radius.

Surface brightness profiles from photomultipliers. The advantage of the large fields of photographic plates, well suited for star counts in the outer parts of

globular clusters, was counterbalanced by the poor spatial resolution which, because of crowding, prevented the resolution of the inner parts — within a few core radii — of most globular clusters. A way out of this dilemma has been the observation of the integrated light, providing surface brightness profiles. This was made possible because of the development of photoelectric devices that measure the surface brightness through different apertures. Photoelectric techniques applied to astronomy were developed, in part, by J. Stebbins and A.E. Whitford (see, e.g., Stebbins & Whitford 1943), and studies of the cores of globular clusters were published, starting in the 1950's by, e.g., Gascoigne & Burr (1956), Kron & Mayall (1960), and more recently by Illingworth & Illingworth (1976), Da Costa (1979), Kron et al. (1984), and Kron & Gordon (1986).

Dickens & Woolley (1967) were the first to employ extensive photometric data with dynamical modelling in their study of ω Centauri. A composite profile made by combining a surface brightness profile for the inner part of the cluster with star counts in the outer part allowed Da Costa & Freeman (1976) to show that single-mass, isotropic King models are unable to fit the entire profile of M3. They generalized these simple models to produce more realistic multi-mass models with full equipartition of energy in the centre.

Star counts and surface brightness profiles from CCD images. It is only with the development, in the eighties, of CCDs (Charge Coupled Devices) for astronomical applications, coupled with software improvement for photometry in crowded fields (e.g., DAOPHOT by Stetson 1987, DOPHOT by Schechter et al. 1993), that the brightest stars in the cores of all globular clusters, even with the highest concentrations, have been at last fully resolved, even in the inner few seconds of arc. The near legendary core of the globular cluster M15 = NGC 7078, which has long been the prototype of the collapsed-core globular clusters, unveiled at least part of its inner structure. The first look at the inner core, with $0.55''$ seeing, was published by Aurière & Cordoni (1981a,b) who partly resolved the three bright central stars. Images with a FWHM resolution of $0.35''$, taken by Racine & McClure (1989) with the High-Resolution Camera of the Canada-France-Hawaii Telescope (CFHT), and, in particular, images with a FWHM of $0.08''$ obtained with the HST, by Lauer et al. (1991) and Yanny et al. (1993, 1994a) with the Planetary Camera, show that most of the former central cusp in luminosity was due to a group of a few bright stars, although post-refurbishment HST data exhibit a star-count profile which continues to climb within $2''$ (Sosin & King 1996, and §9.2).

Globular clusters are known to contain numerous pulsars, bright X-ray sources, and a growing number of dim X-ray sources. Accurate positions are needed for providing possible counterparts to X-ray sources (Paresce et al. 1992; King et al. 1993). The positions of these objects within the clusters can give useful information about their formation process, as well as about mass segregation. Pulsars in clusters can also be used to probe the gravitational potential of the cluster, since changes in the observed period can be attributed to Doppler

shifts induced by gravitational acceleration of the pulsar itself (Phinney 1992). The use of such ways to investigate the internal structure of a globular cluster requires the position of the optical core of the cluster to be well defined. In several cases, particularly for the high-concentration (collapsed) clusters and for the highly obscured clusters towards the galactic centre, the uncertainty in the optical position of the cluster core is now the limiting factor in the determination of the offset between the core and the radio or X-ray position. E.g., Calzetti et al. (1993) report a difference of $6''$ between the positions of the "dynamical" and "light" centres of 47 Tucanae, a difference which is most probably due to the two methods used. The relative accuracy of methods for determining the position of the centres of globular clusters has been investigated by Picard & Johnston (1994) using a testbed of artificial clusters. They also develop a new and more robust method for determining the clusters' centres, giving now positions of the centres with an accuracy of about $1''$ (Picard & Johnston 1996).

Apart from the Galaxy, investigation of the surface brightness profiles of globular clusters has been done so far only in nearby galaxies, like the Large and Small Magellanic Clouds, the Fornax dwarf spheroidal galaxy, and M31. In the Galaxy and M31, the effects of interactions between the clusters and the galaxian central potential, the disk potential, and giant molecular clouds are apparent from the cutoff seen in the radial light and density profiles. The definition of the observed or theoretical tidal cutoff is not a simple issue. The interpretation of the light profile in the outer parts of clusters can depend on assumptions about the isotropy of the velocity distribution and on the net angular momentum of the cluster outskirts (see Weinberg 1993a). The analysis of the profiles consists, most of the time, of a comparison with single- or multi-mass King models, which provides estimates of the core radius, tidal radius, and the concentration. Departures from King profiles are observed, which hamper the quality of the fit and its interpretation (Grillmair et al. 1995a).

Photoelectric, electronographic, and, especially, CCD observations have allowed a systematic investigation of the inner surface brightness profiles of 127 galactic globular clusters and the observational confirmation of the reality of the core collapse phenomenon (Djorgovski & King 1986, Chernoff & Djorgovski 1989). They sort the profiles into two families: (i) the King model clusters and (ii) the collapsed-core clusters. See §9.2 for a general discussion of the use of surface brightness profiles in the study of collapsed-core clusters. A new compilation of basic data and references for 143 galactic globular clusters, along with new deduced King-model structural parameters for 101 of them, are contained in the appendices and tables of the proceedings of the 1992 Berkeley workshop (Djorgovski & Meylan 1993). Trager et al. (1995) present a catalog (available in the AAS CD-ROM series) of surface-brightness profiles of 125 galactic globular clusters, the largest such collection ever gathered, mostly from CCD data. All but four of these surface-brightness profiles have photometric zero points. Central surface brightness, King-model concentrations, core radii, and half-light radii are derived.

The results of two surveys for structural parameters in the surface bright-

ness profiles of young and old clusters in the Magellanic Clouds are given in Meylan & Djorgovski (1987) and Mateo (1987). They emphasize the bumpy surface brightness profiles of the young clusters and mention the possible collapsed character of three old LMC globular clusters, viz., NGC 1916, NGC 2005, and NGC 2019 (see §9.2). Elson et al. (1987a) present the surface brightness profiles of 10 rich star clusters in the LMC, with ages between 8 and 300 $\times$ 10^6 yr. Most of the clusters do not appear to be tidally truncated, and a plausible theoretical interpretation is that expansion of a newly formed cluster either through mass loss or during violent relaxation could lead to the formation of a halo of unbound stars. From calculations including the tidal field of the LMC, they find their clusters extending beyond their tidal radii, with up to 50% of the total masses in unbound halos. In a subsequent study of 35 rich star clusters in the LMC, with ages between 1 Myr and 10 Gyr, Elson et al. (1989) find that the core radii increase from ~ 0 to ~ 5 pc between 1 Myr and 1 Gyr, and then begin to decrease again. They suggest that the expansion of the cores is probably driven by mass loss from evolving stars. See also Elson (1991, 1992).

In contradistinction, the effects on the structure of clusters in the less disruptive milieu of the Fornax dwarf spheroidal galaxy may be visible, providing clues to the initial conditions of the formation of globular clusters and to the extent to which these conditions are mirrored in the structures of those clusters as seen today. See Buonanno et al. (1985b) for the color-magnitude diagrams of all 5 Fornax dSph globular clusters. Rodgers & Roberts (1994) describe the observations of the surface brightness profiles of the five brighter clusters in the Fornax dwarf galaxy. They appears to fall into two groups. Clusters #1 and #2 have similar core radii and follow truncated single-mass King model profiles. Clusters #3, #4, and #5 have similar smaller core radii and extended halos not well fitted by King models. These groupings correlate neither with the differing chemical compositions of the clusters nor with their horizontal-branch morphology, adding further evidence that cluster formation and evolution in Fornax was a complex and diverse process. It is worth mentioning that all five clusters in the Fornax dwarf galaxy are old globulars ($\tau \gtrsim$ 10 Gyr) contrary to the LMC clusters studied in Elson et al. (1989) which have ages between 1 Myr and 10 Gyr. From X-ray imaging of Fornax with ROSAT, Gizis et al. (1993) observe no source in the energy range 10^{36}-10^{38} erg s^{-1}. The low-density environment of the dwarf galaxy evidently does not produce a population of accreting neutron stars through star-star collisions, and no such source is observed in the Fornax dSph globular clusters.

The HST has provided the possibility of studying the surface brightness profiles of globular clusters in the nearby spiral galaxy M31. Bendinelli et al. (1993) and Fusi Pecci et al. (1994) report on the comparison of the structure parameters of M31 globular clusters with those of the galactic globular clusters which shows strong similarities between the two cluster populations.

6.2 Proper motions, stellar radial velocity dispersion, and velocity dispersion from integrated-light spectra

The acquisition of kinematic data brings a great deal of information on the amount and the distribution of mass in globular clusters. These data have been acquired much more recently than surface brightness profiles, since a few technological challenges had to be mastered first: although it is now the case for more than two decades for the radial velocities, it is no yet entirely so for the proper motions (Meylan 1996).

Table 6.1: Internal proper motions in galactic globular clusters

cluster	year	reference
NGC 7078 ≡ M15	1976	Cudworth, AJ, 81, 519
NGC 6341 ≡ M92	1976	Cudworth, AJ, 81, 975
NGC 6205 ≡ M13	1979	Cudworth & Monet, AJ, 84, 774
NGC 5272 ≡ M3	1979	Cudworth, AJ, 84, 1312
NGC 5904 ≡ M5	1979	Cudworth, AJ, 84, 1866
NGC 6838 ≡ M71	1985	Cudworth, AJ, 90, 65
NGC 6656 ≡ M22	1986	Cudworth, AJ, 92, 348
NGC 7089 ≡ M2	1987	Cudworth & Rauscher, AJ, 93, 856
NGC 6712	1988	Cudworth, AJ, 96, 105
NGC 6121 ≡ M4	1990	Cudworth & Rees, AJ, 99, 1491
NGC 6626 ≡ M28	1991	Rees & Cudworth, AJ, 102, 152
NGC 6171 ≡ M107	1992	Cudworth et al., AJ, 103, 1252
NGC 6341 ≡ M92	1992	Rees, AJ, 103, 1573
NGC 5904 ≡ M5	1993	Rees, AJ, 106, 1524
NGC 6656 ≡ M22	1994	Peterson & Cudworth, ApJ, 420, 612
NGC 6121 ≡ M4	1995	Peterson et al., ApJ, 443, 124

Proper motions. In theory, proper motions provide more dynamical information than radial velocities, since they are two- instead of one-dimensional (see, e.g., Wybo & Dejonghe 1995, 1996). The space velocities of some globular clusters are known from radial velocities and absolute proper motions (see, e.g., Cudworth & Hanson 1993); however, the small size of the *internal* proper motions of cluster stars has made them difficult to measure with the required precision. For example, for a nearby cluster at a distance of 5 kpc, a velocity dispersion of 5 km s^{-1} corresponds to a displacement of 20 milliarcsec per century, which is the equivalent of 1.5 micron in 80 years on Yerkes plates. K.M. Cudworth has been the pioneer of this field, squeezing velocity dispersions and

astrometric distances out of the data. But even in the best studied clusters, the errors in the proper motions have been comparable in size to the motions themselves. This explains why only very few studies of cluster internal proper motions have provided dynamical information. Apart from the Cudworth et al. papers in Table 6.1, the only two dynamical studies using published proper motions are, so far, Lupton, Gunn, & Griffin (1987) and Leonard et al. (1992), both of which examined M13 using the data of Cudworth & Monet (1979).

It is obvious that only a very small fraction of the dynamical information contained in globular cluster proper motions has been extracted so far. An expansion and reanalysis of the Yerkes data by Rees (1992, 1993) should result in a welcome increase in the accuracy of the motions and in the understanding of their uncertainties in several clusters. The report on the large proper motion study undertaken by Reijns et al. (1993) for about 7,000 stars in ω Centauri has whetted our appetites, and hopefully will provide essential results in the near future.

Stellar radial velocity dispersion. During the first half of this century and later, all stellar radial velocities were acquired from techniques using photographic plates. The typical errors of the best measurements were $\simeq 10$ km s^{-1}, i.e., of the same order of magnitude, or larger than, the velocity dispersion value expected in globular clusters. A catalog of such radial velocities in galactic globular clusters has been published by Webbink (1981).

But since the pioneering work of Griffin (1967, 1974), cross-correlation techniques have proven their exceptional efficiency in radial velocity determination. The cross-correlation between a stellar spectrum and a template condenses the radial velocity information contained in the stellar spectrum into the equivalent of a single spectral "line", the cross-correlation function. With the construction around the 70's and 80's of instruments using such cross-correlation techniques (e.g., Baranne et al. 1979 and Mayor 1985 for CORAVELs; Flechter et al. 1982, McClure et al. 1985; Latham 1985, Peterson & Latham 1986), the typical errors on the best measurements are $\simeq 0.5$ km s^{-1}, providing an essential tool for investigating the internal dynamics of globular clusters. A new generation of instruments, taking advantage of improved technologies applied to the same cross-correlation techniques, brings the typical errors down to $\simeq 10$ m s^{-1} (e.g., Marcy & Butler 1992, Mayor & Queloz 1995), so far, only for relatively bright ($m_V \lesssim 9$) nearby stars.

The first dynamical study of a globular cluster using high-quality stellar radial velocities was published by Da Costa et al. (1977), who fitted a projected density profile and velocities for 11 giants in NGC 6397 with a single-mass King model. Two years later, Gunn & Griffin (1979) published the first results from their extensive study of cluster velocity dispersions, giving velocities for 111 giants in M3. Density and velocity dispersion profiles were simultaneously fit to multi-mass anisotropic dynamical models based on the King-Michie form of the phase-space distribution function $f(\varepsilon, l)$ (see §7.7 below). Today, eight other clusters have published studies using similar models and sample sizes (between

68 and 469 stars): M92 (Lupton et al. 1985), M2 (Pryor et al. 1986), M13 (Lupton et al. 1987), ω Centauri (Meylan 1987, Meylan et al. 1995), 47 Tucanae (Mayor et al. 1984, Meylan 1988a, 1989), M15 (Peterson et al. 1989 for the velocities and Grabhorn et al. 1992 for the analysis), NGC 6397 (Meylan & Mayor 1991), and NGC 362 (Fischer et al. 1993b).

Stellar radial velocities have been acquired in a few other galactic globular clusters (e.g., Peterson & Latham 1986, Pryor et al. 1989a, 1991), but in smaller quantities, providing weaker dynamical constraints. In the case of M4, Peterson et al. (1995) publish 182 radial velocities with no dynamical study.

Initially, technological developments were driven by the need for small errors in radial velocity measurements ($\simeq 0.5$ km s^{-1}) as required in order to get access to the internal dynamics of globular clusters. The present improvements are now also driven by the size of the samples, as nearly all of the above sets of velocity data are too small to employ, for example, non-parametric methods (see §7.7 below). Acquiring even a few hundred stellar radial velocities one at a time is a slow and tedious job, even on 4-m class telescopes. But the number of stellar velocities in globular clusters has recently grown explosively because of the new technology becoming available to make these measurements. Fiber-fed, multi-object spectrographs like ARGUS at Cerro Tololo, HYDRA at Kitt Peak, and AUTOFIB at the Anglo Australian Observatory can obtain velocities about 25 times faster. Similar gains result from using Fabry-Perot interferometers to measure radial velocities (Gebhardt et al. 1995). Four clusters have published non-parametric studies using sample sizes from a few hundred up to a few thousand stars: 47 Tucanae (Gebhardt & Fischer 1995), NGC 362 (Gebhardt & Fischer 1995), NGC 3201 (Gebhardt & Fischer 1995, Côté et al. 1995), and M15 (Gebhardt et al. 1994, Gebhardt & Fischer 1995). In the framework of the major study of ω Centauri (Reijns et al. 1993), the radial velocities of about 3,500 stars in this cluster have been acquired (Seitzer, pers. comm.) and will be combined with their proper motions.

Stellar radial velocities have been obtained in a few clusters in the Magellanic Clouds (e.g., Seitzer 1991, Mateo et al. 1991, Fischer et al. 1992a,b). Acquiring these stellar radial velocities is difficult because the distance to the Clouds is about ten times larger than that to the best-studied galactic globular clusters. Thus the stars are faint, and crowding and contamination by field stars create serious problems.

Velocity dispersion from integrated-light spectra. The value of the velocity dispersion within $10''$ of the cluster centre is important for the understanding of cluster dynamical evolution, since the velocity dispersion in the core may display a power-law cusp due, e.g., to core collapse. At the same time, this value is very difficult to obtain for high-concentration globular clusters from radial velocities of individual stars because of serious crowding problems. A way to overcome this difficulty is to measure the Doppler broadening in integrated-light spectra obtained from an area of a few square arcseconds at the centre of the cluster.

For globular clusters, the first such observations are those for 10 clusters described in Illingworth (1976), along with a new method based on Fourier power spectra for accurately determining the velocity dispersions. Essentially, this method involves (i) artificially broadening suitable stellar spectra with a range of velocity dispersions (broadening the spectra has the effect of steepening their Fourier power spectra), and (ii) comparing these spectra with the cluster spectrum and selecting the velocity dispersion giving the best match in the Fourier domain.

Fig. 6.1. Cross-correlation functions for five standard stars with very different metallicities (–2.0 $\leq$ [Fe/H] $\leq$ 0.0, from top to bottom, respectively). All cross-correlation functions are (i) well approximated by gaussian functions and (ii) have widths which are independent of the metallicity (from Dubath et al. 1996, Fig. 6).

Since 1987, a numerical version of the analog cross-correlation technique used with CORAVEL spectrometers has been developed at Geneva Observatory (Meylan et al. 1989, Dubath et al. 1990). Instead of doing an analog cross-correlation "online" at the telescope, as is done, e.g., with CORAVEL spectrometers, integrated-light echelle spectra (covering about 1500 Å between

4000 and 7500 Å) are obtained and cross-correlated numerically afterwards. This approach has noticeable advantages: the scanning required to build the CORAVEL analog cross-correlation function at the telescope is no longer necessary, providing an immediate gain of about 2.5 mag, and there are further gains due to the higher quantum efficiency of CCDs as compared to photomultipliers. The read-out noise of the CCD is the limiting factor.

Because the numerical technique has been designed to be similar to the analog technique of the CORAVEL spectrometers (similar templates and wavelength ranges), the numerical cross-correlation functions have the same behavior as the CORAVEL's. CORAVEL experience shows (i) that the cross-correlation functions are well approximated by gaussian functions, and (ii) that the widths of these cross-correlation functions do not depend on the metallicity, as is seen in Fig. 6.1, which displays the cross-correlation functions of 5 standard stars with very different metallicities ($-2.0 \leq$ [Fe/H] ≤ 0.0). Thus, the broadening of a cluster cross-correlation function is only produced by the Doppler line broadening present in the integrated-light spectra because of the velocity dispersion of the stars along the line of sight (Dubath et al. 1996).

Fig. 6.2. Normalized cross-correlation functions of the cluster NGC 1835 (triangles) and of the comparison star HD 31871 (dots). The continuous lines are the corresponding fitted gaussians. The significant broadening of the cluster cross-correlation function is conspicuous and allows an immediate determination of the projected velocity dispersion in the core of NGC 1835 (from Dubath, Meylan, & Mayor 1990, Fig. 3).

An example using the LMC cluster NGC 1835 illustrates the above points (Meylan et al. 1989, Dubath et al. 1990). After normalizing the cross-correlation function of the cluster to have the same depth as the cross-correlation func-

tion of the comparison star, the significant broadening of the cluster cross-correlation function (CCF), $\sigma_{\rm CCF}$(cluster), is conspicuous, as seen in Fig. 6.2. All standard stars have been checked by direct CORAVEL measurements to have almost zero rotation and are used to determine the standard deviation of the instrumental stellar cross-correlation function, $\sigma_{\rm ref}$. Because of the gaussian approximation of the cross-correlation functions, the projected velocity dispersion in the core of NGC 1835 — σ_p = 10.3 ± 0.4 km s^{-1} — is immediately obtained from the following quadratic difference:

$$\sigma_p^2 = \sigma_{\rm CCF}^2(\text{cluster}) - \sigma_{ref}^2. \tag{6.1}$$

In order to study globular cluster masses and mass-to-light ratios as functions of galaxy type and environment, Meylan et al. (1991b) and Dubath et al. (1993b, 1996) have, in the framework of their survey, obtained integrated-light echelle spectra of the core of about 60 galactic, Magellanic, and Fornax globular clusters. Zaggia et al. (1991, 1992a,b, 1993) have developed a similar technique which they applied to seven galactic globular clusters. Dubath et al. (1996) compare their results with those obtained by Illingworth (1976) and Zaggia et al. (1992a,b), for the nine globular clusters which have core velocity dispersion determined by at least two of these three studies. In most cases, for a given cluster, the results are not significantly different from each other (within one sigma). The remaining differences can easily be explained by the differences in sampling areas and cross-correlation techniques. There may be some indications of slight underestimates of errors in a few clusters.

Pryor & Meylan (1993) provide an extensive list of all velocity dispersion data (from individual stars and from integrated-light spectra) available in the literature concerning galactic globular clusters.

In the case of globular clusters, constraints on the velocity dispersion values have been obtained, indirectly, thanks to the presence of pulsars. In dense globular clusters, pulsars are so accelerated by the mean gravitational field of the cluster that their changing Doppler shift can overwhelm the intrinsic positive period derivative $\dot{P}$. The negative $\dot{P}$s provide strict limits to cluster surface mass densities and mass-to-light ratios (Phinney 1992, 1993). Such velocity dispersion estimates come from the use of dynamical models. The two globular clusters M15 and 47 Tucanae contain numerous pulsars with some of them having negative $\dot{P}$s (see Phinney 1993 for M15 and Robinson et al. 1995 for 47 Tucanae).

6.3 Initial and present-day mass functions

During the decades when only photographic data were used, no information was available concerning luminosity and mass functions of globular clusters, apart from the narrow mass range occupied by the giants, subgiants, and the main-sequence stars just below the turn-off. The advent of CCDs, combined

now with the HST, have allowed an increasingly deeper view down the main sequence.

Scalo (1986) gives a review of the early luminosity function results based on photographic photometry of globular clusters. Subsequent deep photometric studies (down to $M_V \simeq 6$) in globular clusters, made possible by CCDs, have shown that the main-sequence luminosity functions vary significantly from cluster to cluster. For example, McClure et al. (1986) tentatively identified, from a sample of CCD-based luminosity functions of 7 globular clusters, a correlation between the cluster metallicity and the main-sequence mass function exponent. However, Richer et al. (1990) and Richer et al. (1991) (see also Richer and Fahlman 1992), find no correlation between the mass function slope and the metallicity. Capaccioli et al. (1991), Capaccioli et al. (1993) and Djorgovski et al. (1993) also find no obvious correlation from an extended sample of 17 galactic globular clusters. They show that, (i) the dispersion in the mass function slopes is much higher than expected from the errors, even after correction for mass segregation effects, and (ii) the position of a globular cluster with respect to the Galaxy acts as a dominant parameter in its mass function slope, while the metallicity plays a weaker role. Capaccioli et al. (1993) interpret this dependence of the mass function slope on the distance from the galactic centre and galactic plane as evidence of a selective loss of stars induced by cluster dynamical evolution. Stiavelli et al. (1991, 1992) show, by using simple semi-analytical models, that the above dependence can be reproduced assuming that all globular clusters are born with identical mass functions, which then evolved through interactions with the Galaxy, pointing towards disk shocking as the most effective phenomenon in stripping the lightest stars. Dauphole et al. (1996) provide calculations of the orbits of 26 galactic globular clusters, which show clearly that some clusters are hit much harder and much more often by passages through the galactic plane then others.

Djorgovski et al. (1993) use appropriate multivariate statistical methods, applied to the sample of 17 galactic globular clusters, to disentangle this complex situation, since the mass function slopes depend simultaneously on more than one variable and many cluster parameters are mutually correlated. They confirm that the mass function slopes in the range $0.5\ M_\odot \leq M \leq 0.8\ M_\odot$ are largely determined by three quantities: mainly the position in the Galaxy (distances to the galactic centre and to the galactic plane, related to the cluster pruning along its orbit), and to a lesser extent metallicity. Their best fitting result gives the following relation for the slope of the global mass function (cf. Eq. 6.3 for the definition of x):

$$x = (3.1 \pm 0.4)(\log R_{GC} + 0.25 \log Z_{GP} - 0.13[Fe/H]) - (3.3 \pm 0.5), \quad (6.2)$$

where R_{GC} and Z_{GP} are the distances in kpc from the galactic centre and plane, respectively. Thus steeper mass functions are associated with clusters which are more distant and/or more metal poor. Other parameters have little effect.

For globular cluster modelling, main sequence stars, white dwarfs and other heavy remnants such as stellar black holes and/or neutron stars have usu-

ally been estimated by simple extrapolation, based generally on the following single power-law form for the whole mass spectrum:

$$dN \propto m^{-x} d\log(m) \tag{6.3}$$

where the exponent x would equal 1.35 in the case of Salpeter's (1955) galactic initial mass function (IMF). There is an irritating ambiguity in the meaning of the phrase "power-law index". Often this refers to Eq. 6.3, but it also often refers to α in the form $dN \propto m^{-\alpha} dm$; note that $\alpha = x + 1$.

Fig. 6.3. HST color-magnitude diagram, summed over the four fields of WFPC2, in the galactic globular cluster M4. Apparent U magnitudes are indicated along the right-hand ordinate, absolute ones on the left-hand. The white dwarf cooling sequence is seen as the bluest stars in the diagram stretching from $M_U \sim +9$ down to the limit of the data near $M_U \sim +13$. This represents the first extensive sequence of cooling white dwarfs seen in a globular cluster (from Richer et al. 1995, Fig. 1).

The presence or importance of stellar remnants and low-mass stars was either ignored or governed by the upper and lower mass limits (typically, m_{sup} = 100 $M_\odot$ and m_{inf} = 0.1 $M_\odot$). The upper limit has no dynamical or photometric influence, because it concerns only small numbers of stars that have already evolved into heavy remnants: e.g., for $x = 1.5$, the fraction of the total mass in the form of heavy remnants varies by 0.05% of the total mass when going from an upper limit of 150 to 50 $M_\odot$; for $x = 1.0$, the same fraction varies by 0.6%, and for $x = 0.5$, by 4.0% of the total mass. The above variations are much smaller than the uncertainty in the total mass. The upper

limit is chosen arbitrarily between 50 and 150 $M_\odot$. The lower limit is much more controversial because of the potential dynamical importance of a large number of low-luminosity stars. There is no observational constraint on the mass function for stellar masses below $\simeq 0.1\ M_\odot$. As noticed by Gunn and Griffin (1979), this lower mass cutoff, if it is low enough, does not significantly affect the cluster structure as traced by the giant stars. Variations of the total mass in the low-mass components do not influence the quality of the fit. The individual mass of the lightest stars is taken generally equal to $0.1\ M_\odot$.

In the case of globular clusters, present-day stellar mass functions may reflect a mixture of both the initial conditions prevailing at the epoch of cluster formation, and the subsequent consequences of the dynamical evolution characterized by a selective escape of stars, i.e., depending on the stellar mass (e.g., King 1996). Consequently, even on the main sequence, there is no way to observe the initial mass function, which has been altered by stellar and dynamical evolution.

The observational constraints related to the initial mass function of stars which were more massive than the present turn-off mass ($\sim 0.8\ M_\odot$) are indirect and vanishingly small. These stars are in the form of dark remnants. Although the bright part of the sequence of white dwarfs is now clearly observed in a few globular clusters (Fig. 6.3), no quantitative parameters (e.g., mass function) can be extracted in order to constrain dynamical models. There is no observational data about the mass function of heavier remnants, like neutron stars, although, in a totally different way, one single pulsar can be used as a dynamical probe of its host cluster (Phinney 1992, 1993).

The HST allows photometry and counting several magnitudes fainter than with ground-based data. For the closest clusters, luminosity functions and mass functions can be determined down to nearly the hydrogen-burning limit. Fortunately, the crude approximations represented by Eq. 6.3 become more and more outdated because of the very deep star counts made possible with HST data (King 1996), providing at last very deep luminosity functions for main sequence stars and white dwarfs (see, e.g., De Marchi & Paresce 1995a, Piotto et al. 1996b, Santiago et al. 1996 for 47 Tucanae; Richer et al. 1995 for M4; De Marchi & Paresce 1994a, Paresce et al. 1995, King et al. 1995, Cool et al. 1996, Piotto et al. 1996b for NGC 6397; De Marchi & Paresce 1995b, Piotto et al. 1996b for M15; and Piotto et al. 1996b for M30).

It should be emphasized that, in most clusters, the mass function is reliably determined observationally only in the interval of about $0.4\ M_\odot$ below the turn-off, i.e., between $0.4\ M_\odot \lesssim M \lesssim 0.8\ M_\odot$. Below $0.4\ M_\odot$ the mass-luminosity relation becomes increasingly uncertain, propagating large errors in the mass function slope for light stars, because of the paucity and faintness of nearby low-mass stars added to the large uncertainties of stellar evolution models, which are in turn due to poor knowledge of stellar opacities. See Henry & McCarthy (1993) for a mass-luminosity relation ($0.08\ M_\odot \leq M \leq 1.0\ M_\odot$) established using a combination of long-term astrometric studies and infrared speckle imaging, and above all D'Antona & Mazzitelli (1996 and references therein) for Population II mass-luminosity relations ($0.09\ M_\odot \leq M \leq 0.8\ M_\odot$)

from stellar models of very low-mass main-sequence stars, with a study of the dependence of the mass-luminosity relation on the metallicity.

Fig. 6.4. HST colour-magnitude diagram (left panel) of NGC 6397, in the filters I_{814} and V_{555}, and the corresponding luminosity function (right panel), compared with the HST luminosity function by Paresce et al. (1995) and the ground-based luminosity functions by Fahlman et al. (1989), from King et al. (1996a, Fig. 1).

In the case of NGC 6397, the two HST-based luminosity functions by Paresce et al. (1995) and by King et al. (1996a) are in good agreement over the common range (Fig. 6.4). Although in agreement at bright magnitudes, at fainter magnitudes, however, the ground-based luminosity function by Fahlman et al. (1989) rises significantly above both the presumably more reliable HST-based luminosity functions. Similar discrepancies, at the faint end of the luminosity function, between HST and ground-based results are noted by Elson et al. (1995) in the case of ω Centauri.

Interesting comparisons are possible between ground-based data from Drukier et al. (1993) and Piotto et al. (1996a) and HST data from Piotto et al. (1996b). The agreement is quite good, suggesting that, while ground-based luminosity functions should not be relied on at very faint magnitudes, they can be relied on at brighter magnitudes. This means that ground- and HST-based data provides nicely complementary information on both ends of the main sequence.

The luminosity functions of M30 and M15 (Fig. 6.5) are very similar, over a range of more than 6 magnitudes, while NGC 6397 is markedly deficient in faint stars. The above data implies that the mass functions of M30 and M15 are very similar in the range $0.12\ M_\odot \leq M \leq 0.8\ M_\odot$. This may be the consequence of both very similar initial conditions and very similar evolution, a scenario which is less contrived than assuming that the present similarity has been created by evolution from different initial conditions. But, since all three

clusters are very similar in metallicity — [Fe/H] $\sim$ –2.0 — and morphology or dynamical status (all are collapsed), what can be the reason why there are so many fewer low-mass stars in NGC 6397 ? The reason is that these clusters have very different orbits around the galactic centre. Although the traditional theories of tidal shocks have never been well enough quantified and are known to be unreliable (Weinberg 1994a,b,c), the effect of tidal shocks is certainly present in the parameters governing the dynamical evolution of globular clusters. NGC 6397, much closer to the galactic plane and galactic centre than M15, is clearly hit much harder and much more often by passages through the galactic plane (King 1996 and Dauphole et al. 1996; see also §10.2).

Fig. 6.5. HST luminosity functions in the filters I_{814} (left panel) and V_{555} (right panel), for three metal-poor collapsed globular clusters, from Piotto et al. (1996b, Fig. 3). The luminosity function of NGC 6397 has been extended up to the turn-off using ground-based data. Where omitted, the error bars are smaller than the symbol size.

It is worth mentioning that ω Centauri is the only cluster, of the five already studied with HST data, for which no drop-off towards fainter luminosities has been detected, to the limit of the existing HST observations (Elson et al. 1995). This may be linked to the strong gravitational potential of this cluster, the most massive galactic globular, which could be about 75 times more massive than NGC 6397 (Drukier 1995, Meylan et al. 1995).

Differences in the radial distributions of stars of different masses are at last definitely observed with HST, providing conclusive observational evidence of mass segregation (see §7.2).

6.4 The possibility of dark matter in globular clusters

This question arises frequently, if only because globular clusters are the next step down in size from the smallest objects in which firm evidence for dark matter can be found, i.e., dwarf spheroidal galaxies (see, e.g., the reviews by Mateo 1994 and Pryor 1994; see also the recent contributions by Armandroff et al. 1995 and Olszewski et al. 1996). A second reason is the theoretical work of Peebles (1984), who showed that an isothermal distribution of stars in a potential dominated by a uniform dark background would have a profile roughly resembling that of a star cluster.

The construction of dynamical models (§7.7)provides one approach to this question. In almost all cases, multi-mass anisotropic King-Michie models do a satisfactory job, and for those cases in which such models are clearly unsuccessful, an interpretation in terms of post-collapse evolution is plausible (e.g., Grabhorn et al. 1992, Phinney 1993). In this sense, then, no dark matter is *required*, except for the modest fractions of neutron stars and white dwarfs included in such models.

One may also ask, how much dark matter *could* there be? It is often found that an adequate fit is obtained with a range of models with a considerable spread of total masses. For example (e.g. Fischer et al. 1992b) found for the LMC cluster NGC 1978 total masses varying over a factor of 5. Though it may be tempting to take this to mean that as much as 80% of this cluster could consist of dark matter, all the models are constructed from ordinary stars and stellar remnants, and it is not clear how much of this could be replaced by dark matter without degrading the fit to the surface brightness. Using simple models, Heggie et al. (1993) and Taillet et al. (1995, 1996) considered how much dark matter could be added before its effects become noticeable (see also Fig.6.6).

The above approaches are open to the criticism that the results are too model-dependent, which has prompted the development of non-parametric methods (see §7.7). In this way Gebhardt & Fischer (1995) have presented quite well constrained estimates of the mass density profiles of several clusters. How much of this is "dark" may be determined, in principle, by using deep star counts (§6.3) to count how much is contributed by normal stars. A preliminary study (Heggie & Hut 1996) suggests that up to half of the inferred mass is invisible. On the other hand it is not implausible that all of this is made up either of white dwarfs (only the brightest of which can be counted at present) or low-mass stars below about $0.1 M_{\odot}$.

New observational techniques for potentially determining the contribution to the mass budget by low-mass stars (which would occupy a halo around the bright stars) are discussed by Taillet et al. (1995). Moore (1996) has recently argued against the existence of such halos of dark matter, pointing out that they would inhibit tidal stripping, in conflict with observation.

Fig. 6.6. Effect of dark matter on a simple model star cluster. The solid line shows the profile of rms projected velocity (upper panel) and surface density (lower panel) in a King model with $r_c = 1$ pc, $r_t = 100$ pc and total mass $10^5\ M_\odot$. The other three curves on each figure show the effect on this "bright" component of adding an equal mass of dark matter, the parameters of the model being adjusted to preserve the mass and scale radii of the bright matter. Short dashes: dark matter particles have same mass as "bright" stars; long dashes: dark matter particles each have 1/8 of the mass of a bright star; dot-dashes: dark matter is uniformly distributed out to the tidal radius.

7. Quasi-static equilibrium: slow pre-collapse evolution

When the phase of violent relaxation (see §5.5 above) comes to an end, a cluster will have settled into a structure close to dynamical equilibrium, except for subsequent transient disturbances as it passes through the galactic plane. In this quasi-equilibrium phase, the cluster will be nearly spherically symmetric if its rotation is slow (as is true of observed clusters in the Galaxy, see §7.6), and if we confine attention to parts well inside the tidal boundary. In this chapter we turn to the dynamical processes which begin to dominate the evolution of a globular cluster in this long phase of existence, where we now find them. Thus this chapter really provides the theoretical background for the remainder of this review, just as the previous chapter provides the observational background.

7.1 The relaxation time

Of the various evolutionary mechanisms we discuss in this section, it is the one which is referred to as "collisional relaxation" or "two-body relaxation" which has the longest history. Long ago Jeans (1929) estimated the time scale on which it acts, and later his result was refined and developed by Chandrasekhar (1942). Genuine collisions are not implied (see §9.4), but rather the purely gravitational encounters of individual pairs of stars. Two stars exchange energy in an encounter, and the cumulative effect of many mild encounters eventually produces major changes in the structure of the cluster, without significantly disturbing its dynamical equilibrium.

The time scale on which this process becomes significant is generally called the *relaxation time*, though several different precise definitions exist. Among theorists the most commonly used is that of Spitzer (1987, Eq. 2-62), who defines:

$$t_r = \frac{0.065\langle v^2\rangle^{3/2}}{\rho\langle m\rangle G^2 \ln\Lambda}. \tag{7.1}$$

Here, $\langle v^2\rangle$ is the mass-weighted mean square velocity of the stars, ρ is the mass density, $\langle m\rangle$ is the mean stellar mass, and $\Lambda \simeq 0.4N$, where N is the number of stars in the cluster.

Eq. 7.1 stems from considering the time scale on which the cumulative mean square value of $\Delta v_{\|}$, i.e., the component of the velocity change which is parallel to the velocity itself, becomes comparable with the mean square value of one velocity component. Other definitions make use, e.g., of the perpendicular component of $\Delta\mathbf{v}$, or the time scale on which the direction of motion of a star is deflected, by two-body encounters, through a large angle. All have a similar form, differing only in the numerical coefficient and/or in the value of Λ (see below). Adequate though these estimates are for many purposes, the time scale of relaxation may be considerably altered by the existence of a spectrum of stellar masses (cf. §7.2 below), or by clumpiness in the spatial distribution

of stars, which may occur in young star clusters (Aarseth & Hills 1972).

Apart from N, the quantities appearing in Eq. 7.1 are local, and so the relaxation time varies from low values in the dense core to extremely large values as the tidal radius is approached. For rough estimates a useful global measure of the time of relaxation substitutes mean values for the inner half of the mass, i.e., within the *half-mass radius* R_h. With one other approximation, based on the virial theorem, which relates the mean square velocity to R_h itself, these considerations lead to the *half-mass relaxation time*:

$$t_{rh} = 0.138 \frac{M^{1/2} R_h^{3/2}}{\langle m \rangle G^{1/2} \ln \Lambda} \tag{7.2}$$

(Spitzer 1987, Eq. 2-63). Values for galactic globular clusters range from about 3×10^7 to about 2×10^{10} years (Djorgovski 1993b).

It is worth relating the relaxation time scale to the other major time scale in the dynamics of star clusters, the *crossing time*. As with the relaxation time this can be defined in several ways, but a common convention is to define:

$$t_{cr} = \frac{2R}{v}, \tag{7.3}$$

where R is a measure of the size of the system and v a measure of the mean stellar velocity. Thus the crossing time is a measure of the time taken for a star to traverse the diameter of the cluster. More specifically, R is often chosen to be the *virial radius*:

$$R_{vir} = -GM^2/(2W), \tag{7.4}$$

where M is the total mass of the cluster and W is its potential energy, i.e., that computed from the interactions among the stars of the cluster (each binary being treated as a single star with a mass equal to the combined mass of the components), and excluding the galactic tidal field. It is often found that R_{vir} is comparable with the half-mass radius R_h; for example, $R_h \sim 0.77 R_{vir}$ in the Plummer model (cf. §7.5). A common specific choice for v is the mass-weighted root mean square velocity of the stars, i.e., $v^2 = 2T/M$, where T is the kinetic energy of the stars (binaries being treated as in the computation of W). With these choices, therefore, the result is that:

$$t_{rh}/t_{cr} = 0.138 \left(\frac{R_h}{2R_v} \right)^{3/2} \frac{N}{\ln \Lambda}. \tag{7.5}$$

Duncan & Shapiro (1982) and Hut (1989) provide instructive introductions to this and other relations between time scales of interest in the internal dynamics of star clusters.

The foregoing estimates are based on the generally accepted theory of relaxation which is described, for example, in Spitzer (1987). On theoretical grounds, however, various modifications or alternatives have been proposed from time to time. The theory is local, as mentioned earlier, and the effects

of this assumption have been discussed by Parisot & Severne (1979) and by Weinberg (1993a). It takes no account of the orbits of the stars in the smooth potential, the effect of which *may* be substantial (Kandrup 1983, Severne & Luwel 1984, Tremaine & Weinberg 1984, and Rauch & Tremaine 1996). The theory also considers only the cumulative effect of many weak interactions, and the effect of the occasional strong interaction requires more elaborate treatment (e.g., Agekian 1959, Hénon 1960b, V'yuga et al. 1976, Retterer 1979, Ipser & Semenzato 1983). The appropriate functional form of Λ in Eq. 7.1 has been questioned on theoretical grounds by Kandrup (1980), with numerical support from Smith (1992), though this contradicted the earlier conclusion of Farouki & Salpeter (1982) (cf. also McMillan et al. 1987). More seriously still, it has been suggested that the combination of relaxation with the chaotic nature of stellar orbits in "non-integrable" potentials (e.g., most axisymmetric potentials) causes a great enhancement in the rate of relaxation (Pfenniger 1986, Kandrup & Willmes 1994). Another suggestion which, if confirmed, would revolutionise the theory of relaxation was made by Gurzadyan & Savvidy (1984, 1986; see also Gurzadyan & Kocharyan 1987, Gurzadyan 1993), and taken up by a number of other authors (e.g., Kandrup 1988, Sakagami & Gouda 1991, Boccaletti et al. 1991). They suggest that relaxation is much faster than in standard theory, by a factor of order $N^{2/3}$. Interestingly, it is claimed that there is support for this view on observational grounds (Vesperini 1992), though Goodman et al. (1993) assert that the time scale estimated by Gurzadyan & Savvidy is wrong and that the mechanism they discuss is not even a relaxation process in the usual sense.

Numerical experiments can provide independent evidence on these debates. Those by Standish & Aksnes (1969) and Lecar & Cruz-Gonzalez (1971) gave results agreeing with those of conventional theory, but the motions of the stars were deliberately simplified. In a much more realistic setting, though with a "softened" potential, Huang et al. (1992) found close agreement between numerical measurements of the "diffusion time" and the relaxation time, and Theuns (1996) has found similar agreement, on the whole, between numerical and theoretical diffusion coefficients. Giersz & Heggie (1993a,b) also found that the results of N-body calculations could be adequately explained by the traditional theory of relaxation, with an appropriate choice of the numerical factor γ in the expression $\Lambda = \gamma N$ for the argument of the Coulomb logarithm (cf. also Giersz & Spurzem 1994, Spurzem & Takahashi 1995, and Fig. 7.1). Any radical revision of the relaxation time scale would destroy their observed consistency between N-body data and conventional theory.

Relaxation affects the evolution of a stellar system in several ways, which are discussed in detail in §§7.2 and 9. In addition, however, it regulates the anisotropy of the distribution of velocities. It is often argued that anisotropy should be small in parts of a cluster where the relaxation time is short, and indeed relaxation can reduce the global anisotropy of a system (Fall & Frenk 1985), but it must also be realised that relaxation by itself can create anisotropy where none was present initially. This has been demonstrated many times, and is the particular topic of studies by Bettwieser et al. (1985) and Bettwieser &

Spurzem (1986).

Another area in which relaxation plays a role is in the rotation of a stellar system. The main information comes from Fokker-Planck simulations by Goodman (1983a) and N-body studies by Fall & Frenk (1985) and Akiyama & Sugimoto (1989). The relation between rotation and escape is discussed in §7.3.

Fig. 7.1. Comparison between four models of the evolution of an isolated stellar system (from Giersz & Spurzem 1994, Fig. 1). The initial model is a Plummer model, and all stars have equal mass. Lagrangian radii (i.e., the radii of spheres containing a fixed fraction of the total mass) are plotted against time. Units are such that G = M = -4E = 1, where M and E are the total initial mass and energy, respectively. Key: AGM – anisotropic gaseous model, IGM – isotropic gaseous model, FOK – isotropic Fokker-Planck model, NBO – average of many N-body models with N = 1,000.

7.2 Energy equipartition and mass segregation

In some theories of star formation, the spatial distribution of stars of different mass will differ at birth (Podsiadlowski & Price 1992, Murray & Lin 1993, Gorti & Bhatt 1996). Usually, however, it is assumed that the processes of

stellar formation give rise to a cluster in which different stellar masses are undifferentiated spatially and dynamically. The early processes of dynamical evolution — mass loss from stellar evolution, and violent relaxation — do not change the cluster in this respect, except for the progressive loss of the more massive stars as they evolve internally. Relaxation is the first dynamical process which does differentiate stars according to their mass. It produces a tendency towards equipartition of kinetic energy, and so the larger mass involved in a gravitational encounter tends to lose kinetic energy, and then fall deeper into the potential well of the cluster. At the same time, stars of lower mass are driven out, and the stars are segregated by mass.

The time scale for this process may be estimated from formulae given by Spitzer (1987, Eq. 2-60), by computing the rate of change of the difference in the kinetic energy of stars in a two-component system. The result is a mass segregation time scale given by:

$$t_{ms} = \frac{0.028(\langle v_1^2\rangle + \langle v_2^2\rangle)^{3/2}}{m_1 m_2 n G^2 \ln\Lambda}, \tag{7.6}$$

where subscripts refer to the two components, and n is the total number density. N-body models show that the time scale for mass segregation (more specifically, for the growth of the half-mass radius of the lighter species) can be well matched by a similar equation (with a suitably chosen coefficient), and it is found empirically that the result can be extended also to continuous mass spectra (Farouki & Salpeter 1982).

If dynamical friction alone is important (which is a satisfactory approximation for the evolution of the stars of greatest mass) the development of mass segregation can be explored with a simplified treatment (White 1976). In general, however, the details of the tendency to equipartition and of mass segregation are best evaluated with the use of a detailed dynamical evolutionary model (see §8 below), and here we summarise the main results in the earlier phases of core collapse (Saito & Yoshizawa 1976; Inagaki 1983, 1985; Inagaki & Wiyanto 1984, Inagaki & Saslaw 1985, Chernoff & Weinberg 1990). Some of these results, however, refer to idealised systems in which stars have only two or a few possible masses, and are obtained with isotropic Fokker-Planck or gas models.

There is first a fairly rapid phase of evolution (presumably on a time scale comparable with Eq. 7.6) in which the different mass components tend towards equipartition in the central regions. How closely they reach equipartition depends on the mass spectrum. Generally speaking, it is most closely approached when either the range of stellar masses is small, or else the spectrum of masses is steep (and so the heaviest stars do not contribute much of the total mass). In other cases there is approximate equipartition amongst the heaviest stars only. These conditions for the achievement of approximate equipartition resemble those derived on the basis of simple theory by Spitzer (1969).

Associated with the (limited) tendency towards equipartition is the process of mass segregation. Just as equipartition tends to be set up only amongst

the heavier masses, the spatial distribution of the heavier masses is greatly differentiated by mass segregation, whereas the spatial distribution of a great range of low-mass stars remains rather similar.

An extreme population for which mass segregation would be important is the population of stellar remnants in the form of black holes of mass $\sim 10M_\odot$ (Larson 1984, Sigurdsson & Hernquist 1993, Kulkarni et al. 1993). Their possible effects on clusters include enhancements of the central velocity dispersion and stripping of the envelopes of red giants, and there is observational evidence for this (Fusi Pecci et al. 1993a).

The foregoing remarks refer to the core. By the time the core has come to equipartition as closely as it ever does, there is still little tendency towards equipartition at and beyond the half-mass radius. This means that observational evidence for mass segregation should be found mostly in the core, and can be obtained by comparison between the core and other regions within the half-mass radius. This may be quite problematic because observational selection effects (crowding and faintness) have similar biases: in the dense crowded core of a star cluster, where stars of low mass should be depleted, faint stars are more easily missed. This makes HST the ideal telescope to look, quantitatively, for mass segregation in globular clusters.

For decades, the differences in the radial distributions of stars of different masses have been seen from the ground, significantly but weakly, in various low-concentration or nearby galactic globular clusters (see, e.g., Sandage 1954 and Oort & van Herk 1959 in M3, Richer & Fahlman 1989 in M71, Drukier et al. 1993 in NGC 6397, among many others). With the HST, however, faint stars can be seen all the way into the core of the clusters, providing strongly significant mass segregation observations. Mass segregation is observed with the HST in NGC 6752 by Shara et al. (1995), in 47 Tucanae by Paresce et al. (1995) and Anderson & King (1996), and, in a more quantitative way, by King et al. (1995, 1996b) in NGC 6397.

In imaging with the HST the high-concentration (core-collapsed) globular cluster NGC 6397, King et al. (1995, 1996b) find the mass segregation effects to be enormous, compared with the marginal degree of segregation observed in this cluster, with ground-based data, by Drukier et al. (1993). Fig. 7.2 displays the mass functions, in stars per arcmin^2, obtained in NGC 6397, at radii $7''$ and $4.6'$, by King et al. (1995). The numbers in the $7''$ field are higher than those in the $4.6'$ field, because of the higher density at the cluster centre, but the mass functions are quite different. Relative to those of high mass, the low-mass stars are depleted at the centre by more than an order of magnitude.

King et al. (1995) have carried out some dynamical modelling to verify that the observed amount of mass segregation is in agreement with dynamical predictions. The use of multimass King models is reasonable here, even though NGC 6397 is a core-collapsed cluster, as long as only very high-concentration models are used (so high that the exact value of the concentration does not matter). Though it is usual to distinguish collapsed from uncollapsed clusters in terms of those which can be fitted with King profiles and those which cannot, this kind of dichotomy refers to *single component* King models. There is no

evidence that fits of *multi-mass* King models to post-collapse clusters are any less satisfactory than those to uncollapsed clusters.

Fig. 7.2. HST mass functions in NGC 6397, at radii $7''$ and $4.6'$, in stars per arcm^2, from King et al. (1995, Fig. 4). The mass functions observed in the $7''$ and $4.6'$ fields are conspicuously different. The solid lines are from a dynamical model fitted to the cluster.

The continuous lines in Fig. 7.2 are from such a dynamical model fitted to the King et al. (1995) observations of NGC 6397: the numbers have been fitted to the observations at $4.6'$ but not at $7''$. The dashed line represents the global mass function of the model. The model is of course chosen to fit the outer points, but there is no requirement whatever that it fit the inner points. The fact that the inner points are indeed reproduced (within the errors) shows that these observations are in satisfactory agreement with theoretical (although somewhat crude) predictions (see also Anderson & King 1996, King et al. 1996b).

7.3 Evaporation through escaping stars

The theoretical study of the rate of escape of stars from clusters has a checkered history, as one sees even from the study of idealised isolated systems. One class of estimates (e.g., Ambartsumian 1938, Spitzer 1940, Chandrasekhar 1942 (his

§§5.3 and 5.4), 1943a,b,c, Spitzer & Härm 1958, King 1965, Danilov 1973, Johnstone 1993) have been based on relaxation phenomena (escape by the cumulative effect of many small disturbances) and yield a fractional rate of escapes proportional to an inverse relaxation time, i.e., $\dot{N}/N \propto -1/t_r$. Another class of theories, based on individual two-body encounters, was developed by Hénon (1960a) and by Woolley & Dickens (1962). These yielded results of a form similar to the first type of method, except for differences in the numerical factor, and the absence of the Coulomb logarithm (which enters in the definition of the relaxation time t_r). Hénon's treatment can be applied conveniently to any system with an isotropic distribution of stellar velocities, and yields a simple analytical result for the Plummer model. He also later tabulated results for a Plummer model in which stars have different masses (Hénon 1969), though his model necessarily excludes mass segregation. Third, a somewhat different method was adopted by Kaliberda (1969), who also treated escape as due to discrete changes in energy rather than diffusion, but considered the same sort of simplified potential as Spitzer & Härm (1958). Finally, a somewhat hybrid theory was presented by Spitzer & Hart (1971a,b), the effect of encounters during one passage through the core being estimated from relaxation theory, and it was applied to other models by Saito (1976).

Detailed modelling is a preferable way of investigating the escape rate, without simplifying assumptions and, in combination with mass segregation, provides the relative escape rates of different stellar masses.

First we summarise some results for isolated systems with equal masses. Though unrealistic, this is an important simplification for understanding the role played by different factors in the escape process. Over a few t_{rh}, modest-sized N-body models (summarised in Wielen 1975) show that $\dot{N}t_{cr} \sim 0.1 - 0.2$, where t_{cr} is the crossing time (Eq. 7.3). Fokker-Planck models (Spitzer & Shull 1975a; cf. §8.2 below) revealed the added refinement that the escape rate increases as the evolution of the system proceeds, at least while the core is still collapsing. Recent N-body models (Giersz & Heggie 1993a) show that this arises from two causes: one is the increasing concentration of the core, and the other is the growth of anisotropy, which tends to enhance the escape rate.

Now we drop the simplifying assumptions which were introduced above. First, in systems of stars with unequal masses, results from theory (Hénon 1969) and N-body models (Wielen 1974a, 1975) show that the overall escape rate (by number) is enhanced, by as much as a factor of 30 for a quite reasonable mass spectrum. Furthermore the rate of escape is heavily mass dependent, the fraction of massive stars which escape in a given time being much smaller than the fraction of low-mass stars. However, there is little difference between the escape rate of stars of lowest mass and those of, say, twice the minimum mass. The fundamental dynamical reason for the mass-dependence of the escape rate is that it is relatively easy for a massive star to impart a large kinetic energy to a low-mass star. This is the same mechanism causing mass segregation, which further depresses the escape rate of massive stars.

The next simplifying assumption to remove is the assumption that the system is isolated, i.e., to reintroduce a steady external field. An important

point to notice with regard to tidally-influenced systems, however, is that the definition of what is meant by "escape" is rather less clear than for isolated systems. If the tidal field is approximated by a spherically symmetric potential then the main effect is that the threshold of escape is lowered, and escape is easier, but no more complicated, than for an isolated system. Even for a cluster in a circular orbit about a spherical galaxy, however, the tidal field is not spherically symmetric (Chandrasekhar 1942, his §5.5), and study of orbits in N-body models (Terlevich 1987) or smooth cluster potentials (Jefferys 1976) shows that it is possible for stars to remain in retrograde orbits bound for long periods to the cluster, even though their orbits take them well beyond the conventional tidal radius. Furthermore, it is only in directions close to those of the Lagrange points that one has a threshold for escape (Hayli 1967); in the orthogonal directions the combined effect of the tidal and centrifugal forces is to help trap stars within the cluster. Ross et al. (1996) have recently established a criterion for escape in this problem (where simple energy considerations are insufficient.)

The relative stability of retrograde orbits has led to the conclusion that a cluster may eventually exhibit substantial retrograde rotation (cf. Oh & Lin 1992). On the other hand various authors (Agekian 1958, Shapiro & Marchant 1976, Longaretti & Lagoute 1996a) have concluded that preferential escape of stars of high angular momentum, which occurs even in the absence of a tidal field, would lead to a *decrease* of rotation and therefore of rotationally induced flattening (if present initially). (Actually Agekian's result was more complicated, as he found that initially highly flattened systems became flatter still.) It should be mentioned that escape is probably not the most effective process for altering the flattening of a rotating cluster, just as it is not the most important process for driving a system into core collapse. In fact Fall & Frenk (1985) found that it is too slow to be of importance, compared with internal processes. Their estimate for the time scale for flattening by internal mechanisms was comparable with that observed in Fokker-Planck models by Goodman (1983a). On the other hand, their study referred to isolated systems of stars of equal mass, and their estimate of the escape time scale was based on a simplified treatment. Their result may, therefore, underestimate the importance of escape. At any rate, it is evident that our understanding of this problem is rather patchy.

In view of these complications, care must be taken in the interpretation of data on the escape rate. Nevertheless, a common approximation is to assume that a star has escaped when its radius exceeds the conventional tidal radius (§7.4), and N-body models show that this leads to consistent results, whether or not a tidal field is included (Giersz & Heggie 1993b). Results from both Fokker-Planck (Spitzer & Chevalier 1973) and N-body models (Hayli 1967, 1970a; Wielen 1968; Danilov 1985; Giersz & Heggie 1996b) confirm that, in systems of stars of equal mass, the presence of a tide greatly increases the escape rate, by about an order of magnitude in the case of the Fokker-Planck models. (A qualitatively different conclusion was, however, reached by Oh & Lin (1992), using a hybrid numerical scheme.)

In systems with a mass spectrum, the loss of stars of low mass is relatively enhanced by mass segregation, which already places these stars at large radii. As with mass segregation itself, however, this does not significantly differentiate the stars of low mass from each other. For example, N-body results (Giersz & Heggie 1996a) and Fokker-Planck results (Chernoff & Weinberg 1990) agree in showing (Fig. 7.3) that, up to the time of core collapse, stars of mass 0.4 $M_\odot$ escape only marginally faster than those of 1 $M_\odot$, in a system with a power law spectrum of masses in the range 0.4 to 15 $M_\odot$. (This result would certainly be altered quantitatively in models including stellar evolution, however.) These simulations dealt with systems up to the point of core collapse; the changes in the evaporation rate in a tidally limited cluster *after* core collapse are described with the aid of Fokker-Planck simulations by Lee & Goodman (1995).

Fig. 7.3. Rate of escape from a model cluster with a steady tidal field but no stellar evolution (from Chernoff & Weinberg 1990, Fig. 13). The initial model was a King model with scaled central potential $W_0 = 3$. Each curve is labelled with the mass in $M_\odot$, the initial mass function being $dN \propto m^{-2.5}dm$, discretised into 16 bins. For each bin the curve plots the remaining fraction of the original mass in that bin, against time in units such that the initial half-mass relaxation time is about 0.019. The collapse time is also about 0.019 unit.

In general terms the preferential loss of stars of low mass from tidally bound systems is further enhanced if the effects of mass loss (e.g., by supernova explosions) are included (Aarseth & Woolf 1972). The grid of Fokker-Planck models computed by Chernoff & Weinberg (1990) is the best starting point for grasping the combined effects of a steady tidal field and mass-loss from stellar evolution. Another time-dependent process which may greatly enhance the preferential escape of low-mass stars is tidal shocking (cf. §6.3, and also Weinberg 1994c).

So far this review of escape has concentrated on clusters with single stars, but a sufficient abundance of primordial binaries (unless these are extremely close) can greatly enhance the rate of *high-velocity* escapers (Leonard & Duncan 1988, 1990). Even in clusters initially consisting of single stars, dynamical processes may lead to the formation of binaries (see §9.5) which then have a substantial effect on the escape rate (Hayli 1970a, Danilov 1978), and especially the flux of energy carried off by escapers (cf. Szebehely 1973, Giersz & Heggie 1994). There is a major difference here between escape due to two-body encounters and that due to binaries; the former actually increases the binding energy of the cluster, whereas the latter causes a decrease. Even in clusters with primordial binaries, the net energy changes due to both process are roughly comparable in magnitude (Heggie & Aarseth 1992). There is also a non-negligible flux of escaping binaries.

Another class of high-energy escapers consists of neutron stars, because of the high space velocities with which they are usually thought to be born. It has been estimated that at most 4% of single neutron stars would be retained within the modest potential well of a typical globular cluster, though larger fractions are retained if the neutron star is a member of a binary which is not disrupted by its formation (Drukier 1996).

In principle the reverse of escape (capture) is possible (Peng & Weisheit 1992).

7.4 Tidal truncation

For many years the emphasis of theoretical studies was on isolated systems, and this is one of several reasons why theoretical work has had less influence on the interpretation of observations than should have been the case. On the other hand the theoretical difficulties posed by inclusion of tidal effects, a few of which are already mentioned in §7.3, are non-trivial.

The motion of a star in a cluster is determined by the potential $\Phi = \Phi_c + \Phi_g$, where the two terms refer to the cluster and the galactic tide, respectively. In an isolated system, Φ_g is taken to be zero. A first non-zero approximation for Φ_g is a spherically symmetric concave function, in which case there is escape of stars outside a certain limiting (tidal) radius R_t at which $d\Phi/dr = 0$. This model leads to the idea of a *cutoff radius*, which is one of the main features of King's models for star clusters, and their derivatives (cf. §7.5). On the other

hand, these models are constructed on the assumption that the potential in which the stars move is Φ_c alone.

A next refinement is to compute more correctly the tidal field experienced by stars in clusters. More precisely, the motion of each star is referred to a frame which moves, like the centre of mass of the cluster, in a smooth orbit in an assumed galactic potential. Then the equations of motion of each star include inertial forces caused by the acceleration of the reference frame, though the relatively small size of clusters justifies the use of a linear approximation for the relative tidal field. Even if Φ_g is steady in an inertial frame, it may be time-dependent in the cluster frame, and the terms "disk shocking" and "bulge shocking" refer to two situations in which this feature is important.

Though relatively unrealistic for globular clusters, the case which can be worked out in some detail is that of a cluster in a circular orbit (e.g., in the equatorial plane of an axisymmetric galaxy; cf. Chandrasekhar 1942, his §5.5). This complicates the construction of models, however, because the tidal field lacks spherical symmetry: as in problems of binary stars, the cluster is surrounded by a Roche lobe with two Lagrangian points (in the directions of the galactic centre and anticentre; cf. Fig. 7.4). Their distance from the cluster is about 1.5 times that of the cutoff radius referred to above (Spitzer 1987, Lee 1990, Heggie & Ramamani 1995). The asymmetry also has an effect on the isotropy of stellar velocities (Oh & Lin 1992), especially at large radii (but within the tidal radius); in addition the Coriolis force tends to deflect stars moving on nearly radial orbits.

If account is taken of the fact that the orbit of a cluster is non-circular, this simple analysis fails. A simple model studied by Angeletti et al. (1983) and Angeletti & Giannone (1983, 1984) showed how the critical mean density for a bound system depended on the eccentricity of the cluster orbit. More elaborate numerical studies (Oh et al. 1992, Oh & Lin 1992) indicate that the cluster is truncated at a radius comparable with the theoretical tidal radius at perigalacticon (as assumed by King 1962 and Ninković 1985), unless the relaxation time is sufficiently short, and then it may be comparable to the theoretical result at apogalacticon. The observational position is not clear (Meziane & Colin 1996).

The time-dependence of the tidal field reaches extreme limits in the cases of disk shocking. Its effects on the orbits of stars was briefly discussed by Keenan & Innanen (1975), using a 3-body model, but its effect on the structure and evolution of the entire cluster was investigated in a series of Fokker-Planck models by Spitzer & Chevalier (1973), and Spitzer & Shull (1975b). For the most part disk shocking has been treated by computing the *mean* change in the energy of a star or cluster using an impulsive approximation, but two recent developments have renewed interest in the process. First, Weinberg (1994a,b) has shown that slow ("adiabatic") disk crossings may be more disruptive than was previously thought. Also, not all effects of the shock are rapidly damped, and, in particular, it may excite an oscillation in which the densest part rocks back and forth within the envelope (Weinberg 1993b); this is reminiscent of the claim by Calzetti et al. (1993) of the offset of the density centre from the

gravitational centre of 47 Tucanae. Second, Kundic & Ostriker (1995) have shown that the energies of the stars in a shocked cluster are subject to *random* changes which act rather like a relaxation mechanism, and it may be especially important for stars beyond the half-mass radius. It has been aptly named "shock relaxation", and it is one factor underlying a recent claim that the rate of destruction of globular clusters in the galaxy has been underestimated (Gnedin & Ostriker 1996).

Fig. 7.4. An N-body model of a cluster evolving in a steady tidal field. Though initially consisting of $N = 8192$ stars, the results are scaled to a cluster with initial mass $1.5 \times 10^5\ M_\odot$, moving at speed 220 $\mathrm{km\,s^{-1}}$ on a circular orbit of radius 4 kpc about a galaxy modelled as a point mass. The model is shown at a time which scales to about 7.7 Gyr, and the unit of length scales to about 10 pc. The initial model was a King model with $W_0 = 5$ and a mass function $dN \propto m^{-3.5}dm$. Mass loss through stellar evolution was included, and about 30% of the mass has been lost by a combination of all processes. The projection into the orbital plane is shown, the horizontal axis being in the direction towards the galactic centre. Stars escape in the vicinity of the Lagrangian points, but are deflected by the Coriolis force. From Aarseth & Heggie (in preparation).

Other important time-dependent tidal processes which have been considered include those due to interstellar clouds (Bouvier 1972; Knobloch 1976, 1977), neighboring stellar systems (Layzer 1977) and hypothetical massive black holes (Wielen 1987). However the disruptive effect of encounters with

interstellar clouds is less important than that of disk shocking (Chernoff et al. 1986). Bulge shocking has been considered by Alladin et al. (1976), Aguilar & White (1985), Spitzer (1987), Weinberg (1994b), and Charlton & Laguna (1995).

From a purely observational point of view, the direct measurement of any tidal or limiting radius is extremely difficult, since it requires the determination of a very low surface density of stars over very large areas with potentially variable back- and foregrounds. In practice, and contrary to all other structural parameters, the determinations of (idealised) tidal radius values for galactic globular clusters, as published, e.g., by Trager et al. (1995) for 125 clusters, are always mere extrapolations of the surface-brightness/star-count profiles, using King (or other theoretical) models.

Grillmair et al. (1995a) provide, in the first study driven by a purely observational approach, two-dimensional surface density maps of the outer structure of 12 galactic clusters (see §6.1 above). The extra-tidal material observed in most of their sample clusters is identified with stars still in the process of being removed from the clusters. The complexity of the structures observed around these clusters illustrates the foregoing theoretical remarks and shows the intrinsic limiting accuracy in the process by which the tidal radius is usually determined.

Chernoff & Djorgovski (1989) find that, in the Galaxy, the distribution of the collapsed-core clusters is much more concentrated about the galactic centre than the distribution of the King model clusters. Within the King model cluster family, a similar trend exists: centrally condensed clusters are found, on average, at smaller galactocentric radii. At fixed distance from the centre, the clusters at smaller heights above the plane (and thus less inclined orbits) are marginally more concentrated. The fact that some internal properties of clusters correlate well with global variables, such as the galactocentric radius, suggests that some external effects are important in cluster evolution.

Another important way in which time-dependent effects relate to tidal truncation is in the dynamics of young star clusters. It is possible that as much as half of the mass of some young clusters in the LMC is in the process of being lost by tidal overflow (Elson et al. 1987a).

7.5 Theoretical models

In this section we assume we are dealing with the dynamics of star clusters at a stage long after the time-dependent effects of the initial conditions have reached dynamical equilibrium. We have seen that time-dependent tides have important effects on the subsequent evolution, and intermittently disturb the assumed equilibrium. Some of these effects may be long-lived, but where disk shocking is mild, which is the case for most galactic globular clusters, it may be expected that departures from dynamic equilibrium will also be slight. Another way of regarding the situation is to observe that the two essential time scales

for dynamical evolution are very different: the crossing time t_{cr}, $\sim 10^6$ yr, is much less than the relaxation time $t_{rh} \sim 10^8$ yr, (and still smaller compared with the typical evolution time $t_{ev} \sim 10^{10}$ yr).

The commonest way of defining a model of a star cluster is in terms of its distribution function $f(\mathbf{r}, \mathbf{v}, m)$, which is defined by the statement that $f d^3\mathbf{r} d^3\mathbf{v} dm$ is the mean number of stars with positions in a small box $d^3\mathbf{r}$ in space, velocities in a small box $d^3\mathbf{v}$ and masses in an interval dm. In terms of this description a fairly general equation for the dynamical evolution is Boltzmann's equation

$$\frac{\partial f}{\partial t} + \mathbf{v}.\nabla_{\mathbf{r}} f - \nabla_{\mathbf{r}}\Phi.\nabla_{\mathbf{v}} f = \frac{\partial f}{\partial t}_{enc},$$

where Φ is the smoothed gravitational potential per unit mass, and the right-hand side describes the effect of two-body encounters. Under the above circumstances, however, the general Boltzmann equation can be greatly simplified. Because t_{cr} is so short, after a few orbits the stars are mixed into a nearly stationary distribution, and so the term $\partial f/\partial t$ is practically equal to zero. In a similar way, because t_{rh} is so long, the collision term $(\partial f/\partial t)_{enc}$ can be ignored. What is left, i.e.,

$$\mathbf{v}.\nabla_{\mathbf{r}} f - \nabla_{\mathbf{r}}\Phi.\nabla_{\mathbf{v}} f = 0, \tag{7.7}$$

is an equilibrium form of what is frequently called Liouville's equation, or the collisionless Boltzmann equation, or the Vlasov equation.

In simple cases, the general solution of Eq. 7.7 is given by Jeans' theorem, which states that f must be a function of the constants of the equations of motion of a star, e.g., the stellar energy per unit mass $\varepsilon = v^2/2 + \Phi$. If not all constants of the motion are known, such functions are still solutions, though not the most general. For a self-consistent solution, the distribution function f must correspond to the density ρ required to provide the cluster potential Φ_c, i.e.:

$$\begin{aligned} \nabla^2\Phi_c &= 4\pi G\rho \\ &= 4\pi G \int m f d^3\mathbf{r} d^3\mathbf{v} dm. \end{aligned} \tag{7.8}$$

Many different kinds of models may be constructed with this approach. In the first place there is considerable freedom of choice over which integrals to include. In the second place one is free to choose the functional dependence of these integrals, i.e., the analytic form of the distribution function (see, e.g., Binney 1982, and Binney & Tremaine 1987, Chapter 4.4). In this section we describe those which are of interest for a variety of reasons, while § 7.7 concentrates on those with important applications in the interpretation of observational data. See also Table 7.1.

Table 7.1: Dynamical models of globular star clusters

		⟵ King	Static Models Michie-King	⟶ 3-Integral	⟵ Gas	Evolutionary Models Fokker-Planck	⟶ N-Body
Dynamical	Features						
Anisotropy		...	√	√	√	√	√
Rotation		...	...	√	...	√	√
Flattening		...	...	√	...	√	√
Dynamical	Processes						
Stellar evolution	1-body	...	...	...	√	√	√
Relaxation	2-body	√	√	√	√	√	√
Tidal Interactions, Collisions	2-body	...	...	...	...	√	√
Stellar Escape	2-body	√	...	...	...	√	√
Primordial Binaries	3- and 4-body	...	...	...	√	√	√
Stellar Motions	collision-less	√	√	√	√	√	√
Steady Tide	collision-less	√	√	√	...	√	√
Disk Shocking	collision-less	...	...	...	...	√	√

Note: under the heading "Dynamical Process", the second column states what kind of physical process it is that is named in the first column

1) Systems whose distribution functions depend only on the energy per unit mass ε. These are the most commonly used models. They are spherical and have an isotropic velocity dispersion ($\overline{v_r^2} = \overline{v_\theta^2} = \overline{v_\phi^2}$):

• Isothermal sphere: this historical starting point cannot itself serve as a realistic model because its density $\rho \propto r^{-2}$ at large radii, which means that the model has an infinite mass. Nevertheless it is of great importance for theory, and is a useful approximation for parts of more realistic models.

• Plummer's model and allied models: Plummer's model is used frequently by theorists for its analytical convenience (cf. Spitzer 1987), but several other sets of analytical models have been investigated (e.g., Bagin 1979).

Veltmann (1983) and Dejonghe (1984) have shown how to construct series of models which include Plummer's model and Hénon's isochrone model (Hénon 1959) as special cases.

• King models: these are the simplest models that observers take seriously. They can be thought of as a modification of the isothermal model (King 1966), with a distribution function given by the "lowered maxwellian" form

$$f \propto \begin{cases} e^{-2j^2\varepsilon} - e^{-2j^2\varepsilon_t} & \text{if } \varepsilon < \varepsilon_t, \\ 0 & \text{if } \varepsilon < \varepsilon_t, \end{cases} \tag{7.9}$$

where j and ε_t are constants. The truncation at energy ε_t corresponds to an absence of very loosely bound stars. The worldwide renown of the King model is probably due to King's ideal combination of both theoretical and observational innovations, supported by a very simple and clear presentation, and ease of computation. By giving results in the observational plane, King's work provided a simple, yet essential, interface between theory and observation. A decade later, da Costa & Freeman (1976) showed that single-mass, isotropic King models are unable to fit the entire density profile of M3. They generalized these simple models to produce more realistic multi-mass models with approximate equipartition of energy in the centre. (The construction of models with equipartition is a non-trivial issue, actually; cf. §7.7 and Merritt 1981). The observational application of King models and their variants is further described below (§7.7). Before passing on from King models, however, it should be mentioned that, despite their equilibrium nature, they have been used to investigate, in a quick but approximate manner, the evolutionary effects of various dynamical processes, assuming that the system evolves along the King sequence. Examples include the studies by Prata (1971a,b), Retterer (1980a), and Chernoff et al. (1986).

• The Wilson sphere and other variants: in this model (Wilson 1975) the distribution function differs from King's distribution in that both the function and its gradient vanish at the boundary ε_t, in contrast to King's distribution which has non-zero gradient at this point. Wilson spheres have more heavily truncated distribution functions than King models, and therefore have more extended envelopes. Other ways of adjusting the maxwellian have been described by Woolley (1954) and Woolley & Dickens (1962), who implemented a cutoff by simply truncating the maxwellian, and by Davoust (1977), Binney (1982) and Madsen (1996).

2) Systems whose distribution functions depend only on the energy per unit mass ε and the specific angular momentum l. Such models are spherical and have an anisotropic velocity dispersion ($\overline{v_r^2} \neq \overline{v_\theta^2} = \overline{v_\phi^2}$):

• Eddington models: Eddington (Shiveshwarkar 1936) took the distribution of the isothermal sphere times $\exp(-j^2l^2/r_a^2)$, where r_a is a constant. This anisotropy factor makes the distribution function almost zero when l is large, i.e., it depopulates all those orbits which at large distances are almost round (see Eq. 7.11). The density profile of an Eddington model falls off more rapidly than that of the equivalent isothermal sphere, but never drops to zero.

• King-Michie models: they associate the "lowered maxwellian" of the King model with the anisotropy factor of the Eddington models (Gunn & Griffin 1979, and Eq. 7.11). They have tidal radii that lie between the tidal radii of the corresponding King and Wilson models. The multi-mass King-Michie models have been the ones most frequently used when fitting simultaneously density and velocity dispersion profiles, and are therefore described further in §7.7.

• There are more such recent models, all of them being variations on the theme of the two integrals of motion ε and l. For example, Osipkov-Merritt models have a distribution function which depends on ε and l only through the variable $Q = \varepsilon - l^2/(2r_a^2)$ (Osipkov 1979, Merritt 1985a,b). Dejonghe (1987) has constructed a convenient series of models which all have the same density profile as a Plummer model, but with varying amounts of anisotropy. Another series described by Louis (1993) has a particularly convenient distribution function. Other examples are given by Batt et al. (1986) and Louis (1990).

3) Systems whose distribution functions depend only on the energy per unit mass ε and the component of angular momentum parallel to the rotation axis l_z. Such models are elliptical and have tangential anisotropy of the velocity dispersion ($\overline{v_r^2} = \overline{v_\theta^2} \neq \overline{v_\phi^2}$). They have been extensively studied in the context of galactic stellar dynamics, where the subject has been efficiently reviewed by de Zeeuw (1987). Observations have shown that rotation is present in globular clusters (§7.6), even though it is weaker than in many galaxies, and this has motivated a renewed search for suitable models.

• Uniform rotation: the simplest distribution functions that involve only ε and l_z have the form $f = F(\varepsilon + \omega l_z)$, where F is an arbitrary function and ω is constant (e.g., Woolley & Dickens 1962, Vandervoort & Welty 1981). In a frame rotating with angular velocity ω the distribution function actually depends on energy alone. Unfortunately, for any distribution function of this form, the mean motion corresponds to rotation at the constant angular speed ω, which is quite unrealistic.

• Prendergast-Tomer and Wilson models: some more realistic models have been introduced by Prendergast & Tomer (1970) and Wilson (1975). The models of Prendergast & Tomer introduce differential rotation not as an explicit part of the theory, but merely as a result of the finite escape velocity interacting with a velocity distribution that would otherwise have yielded a solid-body rotation. In Wilson's models, the differential rotation has been included explicitly via an adjustable parameter. Such models have two characteristics: (i) after an increase in the inner part towards a maximum rotational velocity, the rotation curve decreases towards the outer parts; (ii) the central parts of the model are always rather spherical. These two points make them more suited to globular clusters than elliptical galaxies, although these models have essentially been applied to elliptical galaxies.

4) Systems whose distribution functions depend on a third integral of motion I_3, in addition to the energy per unit mass ε and the component of angular

momentum parallel to the rotation axis l_z. Although no general analytical form for a third integral is available, the existence of an analytic third integral of motion I_3 in special cases has been known for decades, since the work by Jeans (1915). In other cases it has been shown from numerical orbit calculations that the motions of stars are effectively "integrable", which in this context means that they are governed by an approximate third integral. It is because the ellipticities of globular star clusters are so modest that the motions of the stars are unlikely to exhibit any of the usual signs of a breakdown of integrability, such as chaotic orbits.

• 3-integral models: so far, only a few studies have tried to develop 3-integral models; see, e.g., Petrou (1983a,b), Dejonghe & de Zeeuw (1988). The first such study totally devoted to globular clusters (Lupton et al. (1985), Lupton (1985), Lupton & Gunn 1987, Lupton et al. 1987) uses l^2 as a first approximation to I_3, leading to a distribution function depending on ε, l_z, and l^2. Because the rotation creates a nonspherical potential, l^2 is in fact only an approximate integral and Lupton & Gunn's distribution function does not obey the collisionless Boltzmann equation for equilibrium (Eq. 7.7). See Dehnen & Gerhard (1993) for a similar study related to oblate elliptical galaxies.

Models constructed in the way we have described, i.e., by use of Jeans' theorem, are designed to satisfy Eq. 7.7 rigorously. Another approach is to construct models satisfying moments of Eq. 7.7, i.e., Jeans' equations. This is the approach taken by Bagin (1976b) in the construction of multi-component rotating models, and by Davoust (1986) in the single-component case, and this yielded models which he applied to several globular clusters. A hazard of this method is that the resulting model *may* not be realisable using positive distribution functions f (cf. Bagin 1976a).

Stability. One factor which may influence the choice of an appropriate model, whether it is constructed from a distribution function or from Jeans' equations, is its stability. What is at issue here is stability on the crossing time scale, and not on the relaxation time scale; the latter is discussed in §9.1. Thus dynamical stability is concerned with much the same issues as violent relaxation, i.e., whether bulk motions of the matter in a stellar system damp out or grow. This is a large subject, and Merritt (1987a) and Binney & Tremaine (1987, Chapter 5) provide nice introductory accounts. For a full mathematical, but still very readable treatment, Palmer (1994) is recommended. The following remarks describe recent work, especially that related to globular clusters (i.e., spherical or slowly rotating systems).

For spherical non-rotating models, the most relevant instability (Dejonghe & Merritt 1988, Merritt 1990) is the "radial orbit instability", which leads to bar formation in sufficiently anisotropic systems. As shown by Palmer & Papaloizou (1987), this instability can in principle manifest itself in systems in which the global anisotropy is arbitrarily small, and Palmer et al. (1990)

extended this result to axisymmetric systems. In physically more reasonable models, the global anisotropy has to be sufficiently large for the instability to occur (e.g., Weinberg 1991). Saha (1992) has shown how to construct anisotropic spherical models for a single-component system and to test for their stability. Other instabilities affect the radial distribution of the stars without changing the shape of the system (e.g., Stiavelli 1990), or give rise to a displacement of the densest part of the system (e.g., Merritt & Hernquist 1991). Even slowly rotating systems are subject to a “tumbling” instability (Allen et al. 1992).

7.6 Observational evidence of cluster rotation

Compared with galaxies, galactic globular clusters are anomalously spherical in shape. The flattest elliptical galaxies observed are of type E7, in striking contrast to the flattest galactic globular clusters, which have type E2. As relaxation times of galaxies greatly exceed the Hubble time, a quasi-steady evolution cannot have altered substantially either the initial dynamical structure or the shape of such stellar systems. On the contrary, central relaxation times t_{rc} of globular clusters being much shorter than the age of the universe (typically $10^6 \lesssim t_{rc} \lesssim 10^8$ yr; frequently $t_{rh} \lesssim 10^9$ yr), strong evolutionary changes may have transformed the shape of the globular clusters since the time of formation.

Agekian (1958), Shapiro & Marchant (1976), and Longaretti & Lagoute (1996a) all studied the way in which the angular momentum carried off by escapers can affect the ellipticity (see §7.3 above), and found that the ellipticity can decrease significantly over the lifetime of a cluster. An age-ellipticity relation has indeed been observed by Frenk & Fall (1982) for clusters in the Galaxy and in the Large Magellanic Cloud (see also Geyer et al. 1983 and Akiyama 1991).

An interesting study of the true shape of globular clusters is given in Fall & Frenk (1983), who discuss the distributions of true and apparent ellipticities for random orientations. The intrinsic shapes of globular clusters in our Galaxy, M31, and in the Large and Small Magellanic Clouds are compared by Han & Ryden (1994; see also Ryden 1996). They find that, for the galaxies with similar structure, mass, and age, their globular clusters tend to have similar shapes, i.e., the clusters in our Galaxy and M31 are, on average, more spherical than those in the Magellanic Clouds. It is worth mentioning that, due to the intrinsic differences between photographic and CCD images and between the various ellipticity estimate techniques and definitions (e.g., mean ellipticity or at a given radius), the measured ellipticities for individual clusters frequently disagree.

The detection of rotation in globular clusters (from proper motions and/or radial velocities of numerous individual stars) suffers always from the uncertainty due to the lack of knowledge of sin i, where i is the angle between the line of sight and the rotation axis of the cluster ($i = 90°$ when equator-on, $i = 0°$ when pole-on).

Rotation from stellar proper motion measurements. So far, it is only in the case of M22 that Peterson & Cudworth (1994) have been able to clearly detect rotation from proper motion data, although Rees & Cudworth (pers. comm.) see rotation in the 47 Tucanae proper motions as well.

Table 7.2: Rotation in galactic globular clusters

cluster	V_{rot}^{max} ($\mathrm{km\,s^{-1}}$)	V_{rot}^{max} / σ	ε (1)	number of stars	reference
NGC 5272≡ M3	1.0	0.12	0.04	107	Gunn & Griffin (1979)
NGC 6341≡ M92	2.5	0.30	0.10	49	Lupton et al. (1985)
NGC 7089≡ M2	5.5	0.34	0.11	69	Pryor et al. (1986)
NGC 5139≡ ω Cen	8.0	0.32	0.17	318	Meylan & Mayor (1986)
NGC 104 ≡ 47 Tuc	6.5	0.26	0.09	272	Meylan & Mayor (1986)
NGC 6205≡ M13	5.0	0.25	0.11	142	Lupton et al. (1987)
NGC 6397	0.5	0.11	0.07	127	Meylan & Mayor (1991)
NGC 6656≡ M22	3.8	0.50	0.14	130	Peterson& Cudworth (1994)
NGC 362	0.0	0.01	0.01	208	Fischer et al. (1993b)
NGC 7078≡ M15	1.7	0.15	0.05	216	Gebhardt et al. (1994)
NGC 3201	1.2	0.28	0.12	399	Côté et al. (1995)
NGC 5139≡ ω Cen	7.9	0.41	0.17	469	Merritt et al. (1996)

(1) all ellipticity values from White & Shawl (1987)

Rotation from stellar radial velocity measurements. Cross-correlation techniques provide stellar radial velocities with errors of typically 1 $\mathrm{km\,s^{-1}}$ or less, i.e., significantly smaller than proper motion uncertainties, and have, consequently, allowed detection of rotation in a few globular clusters (see Table 7.2).

The galactic globular cluster in which rotation is expected the most is ω Centauri, the giant southern globular cluster, which has the largest mean ellipticity $\langle\varepsilon\rangle = 0.12$ (with $0.05 \leq \varepsilon \leq 0.17$, Geyer et al. 1983) and the longest central relaxation time ($t_{rc} \simeq 10^9$ yr, Meylan 1987). The first indication of the presence of rotation is published by Harding (1965), who uses a sample of 13 stars (each having at least three radial velocity measurements), with the projection of the rotation axis supposed identical to the minor axis of the stellar distribution on the plane of the sky. An unpublished study by Seitzer (1983), based on 118 stars, displays the differential rotation of ω Centauri.

In ω Centauri Meylan & Mayor (1986) use the radial velocities of 318 stars, scattered on the plane of the sky between $0.30'$ and $22.4'$ from the cluster centre. These data reveal immediately the presence of rotation by analysis of the radial velocities according to the hypothesis of a projected differential rotation of the type:

$$\langle V_r \rangle = A(r)\sin(\alpha + \phi) + V_\circ \tag{7.10}$$

where r and α are the polar coordinates of the star with respect to the cluster centre. This approach has been used in most of the studies mentioned in Table 7.2. Fig. 7.5 displays a plot of V_r vs. α, where V_r is the mean radial velocity of each star, for all stars measured in ω Centauri in a ring on the plane of the sky in which the maximum of the rotation is reached. The sinusoidal distribution of the points betrays immediately the presence of rotation, although Eq. 7.10 does not describe the real behavior of the field of radial velocities as projected on the plane of the sky.

Fig. 7.5. Radial velocities V_r as a function position angles α, for 205 stars in ω Centauri with radii r between 300$''$ and 1200$''$ (from Meylan & Mayor 1986, Fig. 1a). The conspicuous sinusoidal distribution of the points reveals the presence of rotation.

In order to estimate the systemic rotation of the cluster as a whole around the axis of symmetry of the ellipsoid, Meylan & Mayor (1986) use an ad hoc analytic form, as general as possible, in order to mimic any kind of rotation curve (e.g., flat or Keplerian). This has the advantage of reducing the dependence on any idiosyncrasies of the model. The analytic form depends on four free parameters. Three of them describe the equatorial rotation curve, namely (i) a solid rotation in the cluster inner part, (ii) the maximum of the rotation curve V_{rot}^{max} and its distance r_{max} from the axis of symmetry, and (iii) a differential rotation in the outer parts; a fourth parameter represents (iv) the decrease of the rotation in the direction of the poles, since the cluster

does not have cylindrical rotation. For each nonlinear least-squares fit between computed and observed velocities, sin i ($0° \leq i \leq 90°$) is a fixed parameter. Rotation is definitely observed in ω Centauri. For $i = 90°$, $V_{rot}^{max} = 8.0$ km s^{-1}, occurring at 3-4 r_c. This non-cylindrical differential rotation is most important in a central torus and weak in the outer parts (see Fig. 7.6). The angular velocity Ω_c inside 1 r_c equals 1.4×10^{-6}yr^{-1}, which corresponds to one revolution of the core in 4.5×10^6 yr. The similarity between the rotation and the ellipticity curves is impressive and suggests that the flattening of ω Centauri is due to rotation (see Fig. 2a in Meylan & Mayor 1986).

Fig. 7.6. Line-of-sight rotational velocity field in the first quadrant of the meridional plane, from a non-parametric estimate of the rotation in ω Centauri by Merritt et al. (1996). The units on both axes are arcminutes and contours are labelled in km s^{-1}.

With an enlarged sample of 469 stellar members of ω Centauri (Meylan et al. 1995), a non-parametric (see §7.7) estimate of the mean line-of-sight velocity field on the plane of the sky has been constructed by Merritt et al. (1996). Feast et al. (1961) have shown that some of the observed rotation is merely a perspective effect caused by the proper motion of the entire cluster. Merritt et al. (1996) have corrected for this slight effect (about 1 km s^{-1} in the case of ω Centauri). Fig. 7.6 displays the line-of-sight rotational velocity field in the first quadrant (Merritt et al. 1996). These results confirm, both qualitatively and quantitatively, those displayed in Fig. 3 in Meylan & Mayor (1986) and obtained in a completely different way.

Mayor et al. (1984) provides the first detection of rotation in 47 Tucanae,

the second best studied globular cluster, for which the mean ellipticity $\langle\varepsilon\rangle$ = 0.10 (with $0.08 \leq \varepsilon \leq 0.13$, Geyer et al. 1983). Meylan & Mayor (1986) use mean radial velocities of 272 member stars scattered on the plane of the sky between $0.15'$ and $14.4'$ from the cluster centre. In a way similar to the study of ω Centauri, rotation is definitely observed in 47 Tucanae. For $i = 90°$, V_{rot}^{max} = 6.5 km s^{-1}, occurring at 11-12 r_c. This non-cylindrical differential rotation is most important in a central torus and weak in the outer parts. The angular velocity Ω_c inside 1 r_c equals 2.7×10^{-6}yr^{-1}, which corresponds to one revolution of the core in 2.9×10^6 yr. Probably because of poor ellipticity data, the similarity found in ω Centauri between the rotation and the ellipticity curves is not observed in 47 Tucanae.

All the above results obtained for ω Centauri and 47 Tucanae depend on the value of the angle i, which remains unknown. Since these two clusters belong to the small group of the clusters which, among the 150 galactic globular clusters, are the flattest ones, we can expect, from a statistical point of view, that their angles i should not be very different from $60° \leq i \leq 90°$. The importance of rotation (for a given projected rotation velocity) increases as i gets closer to 0°. The relative importance of rotational to random motions is given by the ratio $V_\circ/\sigma_\circ$, where $V_\circ^2$ is the mass-weighted mean square rotation velocity and $\sigma_\circ^2$ is the mass-weighted mean square random velocity. For i = 90° and 60°, in ω Centauri the ratio $V_\circ/\sigma_\circ = 0.35$ and 0.39 and in 47 Tucanae the ratio $V_\circ/\sigma_\circ = 0.40$ and 0.46, respectively (Meylan & Mayor 1986). Even with $i = 45°$, the dynamical importance of rotation remains weak compared to random motions. The ratio of rotational to random kinetic energies is $\simeq 0.1$, confirming the fact that globular clusters are, above all, hot stellar systems.

As displayed in Table 7.2, rotation has been directly observed and measured in ten globular clusters. The diagram $V_\circ/\sigma_\circ$ vs. $\langle\varepsilon\rangle$ has been frequently used for elliptical galaxies and its meaning is extensively discussed in Binney & Tremaine (1987 Chapter 4.3). The low luminosity ($L \lesssim 2.5\ 10^{10}\ L_\odot$) elliptical galaxies and spheroids have ($V_\circ/\sigma_\circ$,$\langle\varepsilon\rangle$) values which are scattered along the relation for oblate systems with isotropic velocity-dispersion tensors, while the high luminosity ($L \gtrsim 2.5\ 10^{10}\ L_\odot$) elliptical galaxies have ($V_\circ/\sigma_\circ$,$\langle\varepsilon\rangle$) values which are scattered below the above relation, indicating the presence of anisotropic velocity-dispersion tensors. Given their small ellipticities ($0.00 \leq \langle\varepsilon\rangle \leq 0.12$), globular clusters are located in the lower-left corner of the $V_\circ/\sigma_\circ$ vs. $\langle\varepsilon\rangle$ diagram, an area characterized by isotropy or mild anisotropy of the velocity-dispersion tensor.

7.7 Model fitting: parametric and non-parametric methods

During the last two decades, the purpose of building dynamical models has been to construct various simplified mathematical descriptions of a star cluster, each of them easily comparable with observations. The general principles on which such models may be built are described in §7.5, where many examples were

summarised. Very few of these, however, have achieved any prominence in applications, and it is on these that we concentrate in the present subsection. Foremost amongst these are King's models and their variants.

King models approximately incorporate three essential dynamical processes (Table 7.1): (i) dynamical equilibrium, (ii) the effect of gravitational encounters between pairs of stars, which, like collisions in a gas, tend to set up a maxwellian distribution of velocities, and (iii) a cutoff in energy (ε_t, cf. Eq. 7.9) above which stars are deemed to have escaped; the cluster potential Φ_c takes this value at a finite radius, which can be interpreted loosely as the tidal radius r_t (§7.4). (As already mentioned in §7.4, however, in the construction of King models, the galactic potential Φ_g is ignored.) The models are readily constructed numerically, and results are conveniently tabulated in Ichikawa (1985).

King's models depend on three dimensional parameters, which can be taken to be the central density ρ_c, the central velocity dispersion σ_c, and the tidal radius r_t. There is one dimensionless parameter, which can be taken to be the ratio of the tidal to core radius, i.e., the concentration $c = r_t/r_c$, or the dimensionless central potential (Fig. 7.3). The definition of r_c can cause confusion, but here it refers to the scaling length which appears in the theory of the models, which satisfies $8\pi G\rho_c r_c^2 j^2/9 = 1$. This is often extended to other models by replacing $3/(2j^2)$ by the central three-dimensional velocity dispersion, but even for King models this is only an approximation. There is no really satisfactory theoretically-based definition for multi-component models.

As originally described, King models are single-component models, i.e., Eq. 7.9 makes no distinction between stellar masses. The construction of multi-mass variants is a matter of reasoned choice. If the analogy is made with the kinetic theory of gases, one assumes that the constant j^2 in Eq. 7.9 is proportional to mass (e.g., Illingworth & King 1977, Kondrat'ev & Ozernoi 1982). In a maxwellian distribution this leads to the usual equipartition of kinetic energies, but with the lowered maxwellian used in King models, the distribution of velocities becomes nearly independent of mass for the lowest masses. The specification of the mass function is the usual compromise between convenience and realism. It is often taken to be a power law (see Eq. 6.3).

After the introduction of a mass spectrum, the next important variant of King's original models deals with the isotropy of the velocity distribution. When Eq. 7.9 is used for the distribution function, the distribution of velocities is isotropic. In order to include anisotropy, the distribution function can be made to depend on the specific angular momentum l. Following Gunn & Griffin (1979), the most commonly chosen form is

$$f \propto \exp(-j^2 l^2/r_a^2)(\exp(-2j^2\varepsilon) - \exp(-2j^2\varepsilon_t)) \qquad (7.11)$$

for $\varepsilon < \varepsilon_t$, where r_a is a constant. This introduces a second dimensionless parameter (the ratio of r_a/r_c, for example). When anisotropy is introduced in this way along with unequal masses, it is usual to take r_a to be the same for all masses, as it corresponds to the radius beyond which the anisotropy becomes important. Incidentally, the increase of anisotropy with radius may not be all

that appropriate for tidally bound models, because of the isotropising effect of the tidal field, which is important at large radii.

The construction of King-Michie models and their variants is based on a simplified mathematical description of a star cluster. Such models allow the quick computation of grids of models which sample large ranges for the values of the free parameters which are, typically, the central potential, the mass function index, and the anisotropy radius. The fits of these models to the observational constraints (generally, density and velocity dispersion profiles) provides the structural parameters, i.e. the radii r_c, r_h, and r_t, and hence the concentration c, along with stellar density, total mass, and M/L_V values. The most updated list of structural parameters is given by Trager et al. (1995) for 125 galactic globular clusters.

The studies constrained simultaneously by density and velocity dispersion profiles provide the most reliable estimates of stellar densities (the central mass density $\rho_\circ$, the mean mass density ρ_h inside the half-mass radius, and the mean mass density ρ_t inside the tidal radius), with the total mass of the cluster and its central and global M/L_V. Table 7.3 gives the list of the main studies using multi-mass King-Michie type models simultaneously constrained by density and velocity dispersion profiles. In these studies, all radial velocities have been acquired by single-object spectrometers.

Table 7.3: Dynamical studies using King-Michie type models

cluster	authors	reference
M3	Gunn & Griffin	1979, AJ, 84, 752
47 Tucanae	Mayor et al.	1984, A&A, 134, 118
M92	Lupton et al.	1985, IAU Symp. 113, p. 327
M2	Pryor et al.	1986, AJ, 91, 546
M13	Lupton et al.	1987, AJ, 93, 1114
ω Centauri	Meylan	1987, A&A, 184, 144
47 Tucanae	Meylan	1988, A&A, 191, 215
47 Tucanae	Meylan	1989, A&A, 214, 106
M15	Peterson et al.	1989, ApJ, 347, 251
NGC 6397	Meylan & Mayor	1991, A&A, 250, 113
M15	Grabhorn et al.	1992, ApJ, 392, 86
NGC 362	Fischer et al.	1993b, AJ, 106, 1508
NGC 3201	Da Costa et al.	1993, ASP Conf. Ser. 50, p. 81
ω Centauri	Meylan et al.	1995, A&A, 303, 761

The most updated list of dynamical parameters, like density, mass and M/L_V values, is given by Pryor & Meylan (1993) for 56 galactic globular

clusters. It is often said that such total masses and global M/L_V values are very model-dependent, partly, it is argued, because the observations poorly constrain very low-mass populations. In fact, however, it was pointed out by Gunn & Griffin (1979) that replacing N stars of low mass m by, say, $2N$ stars of mass $m/2$ has little effect on the model fits, since mass segregation implies that most of these stars are at large radii, almost independent of their precise mass. In addition, it is argued that surface brightness and velocity dispersion profiles do not uniquely determine the cluster mass. Nevertheless the basis of the non-parametric methods advocated by Merritt (see below) is that they *do* constrain the mass distribution uniquely, provided the distribution function is isotropic. Therefore it is the degree of anisotropy which may well be the most important model-dependent assumption, and little is known about its effect on the inferred total masses.

Using biweight estimators (Beers et al. 1990; these estimators are insensitive to outliers) with the entire sample of 56 clusters yields a mean M/L_V of 2.3 and a dispersion about the mean of 1.1. Similarly, the mean central M/L_V is 1.7 and the dispersion is 0.9. The global M/L_V does not correlate significantly (absolute value of the correlation coefficient $|r| < 0.22$) with distance from the galactic centre, distance from the galactic plane, metallicity, concentration, or half-mass relaxation time. Global M/L_V is weakly correlated with the total mass ($r = 0.31$). Independence of these two quantities can be rejected at better than 95% confidence, but this conclusion and the correlation coefficient are compromised by the correlation between the errors in mass and M/L_V. Mandushev et al. (1991) have found similar results with a sample of 32 clusters. Whether M/L_V really tends to increase with increasing mass is still uncertain (Pryor & Meylan 1993).

Table 7.4: Dynamical studies using non-parametric techniques

cluster	authors	reference
M15	Gebhardt et al.	1994, AJ, 107, 2067
"	Gebhardt & Fischer	1995, AJ, 109, 209
47 Tuc	Gebhardt & Fischer	1995, AJ, 109, 209
NGC 362	Gebhardt & Fischer	1995, AJ, 109, 209
NGC 3201	Gebhardt & Fischer	1995, AJ, 109, 209
"	Côté et al.	1995, ApJ, 454, 788
ω Centauri	Merritt et al.	1996, submitted

The purpose of building models such as King's models and their derivatives is to construct a simplified mathematical description of a star cluster. They provide a number of parameters (mass spectrum, concentration, anisotropy radius, etc.) which can be adjusted to optimise the fit with observations. Nevertheless they are based on strict assumptions with regard to the form of

the distribution function. These assumptions take account of dynamical theory, though in some respects they contradict or oversimplify it. The results can be strongly biased by the choice of the integrals of motion and the form of the functional dependence. Indeed it is true that, even when the profiles of mass density and velocity dispersion are known, the distribution function is still not uniquely determined (Dejonghe 1987, Dejonghe & Merritt 1992); for instance the anisotropy is still not completely constrained. For this and other reasons it is worth considering methods which attempt to construct the distribution function from the observations, with minimal assumptions.

There are quadratic programming techniques (Dejonghe 1989) which fall into this class. The distribution function is written in terms of basis functions, but the possible risks of such an approach have been discussed by Merritt & Tremblay (1994) in a related context. They recommend a modification which serves to smooth the resulting estimate of the distribution function. The general aim is to infer the gravitational potential $\Phi(r)$ and the phase-space distribution function $f(\varepsilon)$, given the observations of the surface density and velocity dispersion profiles of a "tracer" population. Briefly, in the case of a globular cluster, (i) the projected density $I(R)$ provides the space density $\nu(r)$, (ii) the projected velocity dispersion $\sigma^2(R)$ provides the space velocity dispersion $v^2(r)$, (iii) the Jeans equation provides the gravitational potential $\Phi(r)$, and (iv) the Eddington equation provides the phase-space distribution function $f(\varepsilon)$. Nevertheless, a disadvantage of such techniques arises from the delicate process of deprojection using Abel integrals (Merritt 1993a,b,c).

Table 7.4 gives a list of the non-parametric studies which have been published for five globular clusters, using samples from a few hundred up to a few thousand stars. As a result, non-parametric mass density and M/L_V profiles can now be compared with more traditional theoretical models for core-collapse clusters. The two non-collapsed globular clusters, viz., NGC 362 and NGC 3201, seem to exhibit significant differences from the two possibly collapsed globular clusters, 47 Tuc and M15. The derived phase-space distribution functions are not consistent with King models: NGC 362 and NGC 3201 have significantly more tightly-bound stars than King models, and systematic differences appear between 47 Tuc and M15 and either the King models or the two less concentrated globular clusters. E.g., Fig. 7.7, which displays the non-parametric estimates of the M/L_V for the four clusters NGC 362, NGC 3201, M15, and 47 Tucanae, shows a remarkable difference between the M/L_V profiles of the two collapsed and the two other clusters. Côté et al. (1995), using King, King-Michie, and non-parametric models, present, for NGC 3201, an interesting comparison between the different results, and a discussion of how to disentangle the consequences of the assumptions and disadvantages of each approach.

Incidentally, these methods do not attempt to construct a distribution function for the entire cluster, but only one for a stellar species for which both positional and kinematic data are available, and the gravitational potential. A possible criticism of this technique is that it goes too far in entirely ignoring dynamical theory, except Jeans' theorem, and takes no account of the fact that

the inner parts of all galactic globular clusters should be nearly relaxed. Also it has not so far been extended to the construction of models with anisotropic distribution functions, though axisymmetric systems can now be treated (Merritt 1996).

Fig. 7.7. Non-parametric estimates of the M/L_V (solid lines) and their 90% confidence bands (dotted lines), from Gebhardt & Fischer (1995 Fig. 7). The two possibly collapsed globular clusters, 47 Tuc and M15, differ significantly from the two non-collapsed ones, NGC 362 and NGC 3201.

8. Evolutionary models

The models described in the previous sections (§§7.5 and 7.7 especially) take account of a certain amount of dynamics, mainly the assumption that the cluster is in dynamic equilibrium (Jeans' theorem). To some extent, but always approximately, some of these models also take into account the effects of gravitational encounters. The methods described in the present section are,

however, needed if the effects of these processes are to be modelled with any precision. They can also incorporate a broad spectrum of important processes which influence both the gross evolution of a cluster and the evolution of its individual components (cf. Hut et al. 1992a for an interesting general review).

8.1 N-body integrations

Ideally, the dynamical evolution of a globular cluster would be modelled with a direct N-body integration. In fact this method is inapplicable to globular clusters, because the required value of N (of order 10^6) is too large for a simulation to be completed within a reasonable time (Fig. 8.1). Larger values are used in cosmological N-body simulations, but there one may exploit approximations in the evaluation of forces which would lead to unacceptable errors in the simulation of a star cluster, and the required number of time steps is much smaller. Similar simplifications can be adopted in simulations of star clusters for the investigation of certain kinds of phenomena, namely, those that do *not* involve two-body relaxation effects, close encounters between binaries, etc., and here the fast methods that have been developed for the study of galaxy dynamics may be employed. This is too wide a subject for review here, and the following paragraphs are devoted to N-body techniques which can faithfully model all the gravitational processes that are relevant in globular clusters. Despite the limitation on N, direct N-body simulations can teach us much about the dynamical evolution of globular clusters, provided that the scaling with particle number is understood. Many developments have taken place since the review by Aarseth & Lecar (1975).

At present the best code is NBODY5 (Aarseth 1985a), and descendents which are still under development. In addition to a high-order integrator (similar, in early versions, to that described in Wielen 1974b), it incorporates a number of subtle techniques which are indispensable for adequate accuracy and efficiency, including the use of individual time steps (so that the positions and velocities of different particles are advanced with different frequencies), computation of forces from near neighbors and distant stars with different frequencies (the scheme of Ahmad & Cohen 1973), and special treatments (regularisation) of compact pairs (binaries) and other few-body configurations (Mikkola 1985, Mikkola & Aarseth 1990, 1993). Modelling of star clusters with primordial binaries, for example, would be impractical without these techniques. Even so, the simulation of a cluster with only a few thousand stars and a few percent of primordial binaries takes about 2,000 hours on a typical workstation (Heggie & Aarseth 1992).

Though the mix of techniques used in such codes as NBODY5 is well tried and successful, it is always possible that improvements remain to be discovered. Makino (1991), for example, has examined such aspects as the time step criterion and order of the integration routine (cf. also Wielen 1967). Other integration schemes have been considered (Mann 1987, Press & Spergel 1988);

relatively recently it has been found that so-called "Hermite" integration techniques (which have since been incorporated into NBODY5) can offer significant advantages (Makino & Aarseth 1992), and new life has been breathed into the humble leapfrog integrator by Hut et al. (1995); see also Funato et al. (1996) for an application of the same ideas to regularisation. The leapfrog method is a so-called "symplectic integrator", which refers to a class of methods which have some attractions in studying N-body problems without dissipation, and there has been considerable activity in this area (e.g., Ruth 1983, Forest & Ruth 1990). In principle it is actually possible to express the solutions of the N-body problem as infinite series (Wang 1991), but so far this has not proved a useful guide to new numerical methods.

Fig. 8.1. The progress of N-body simulations. Each plotted point gives the date of publication of the largest N-body simulation at that time which extended well into core collapse (at least), except for the last two points, which refer to preprints. The largest value of N has increased by almost one decade per decade.

Indirect methods of evaluating forces (tree or hierarchical schemes: Appel 1983, Barnes & Hut 1986, 1989, Ambrosiano et al. 1988, Hernquist 1988, and especially McMillan & Aarseth 1993) should also become of increasing importance as the feasible values of N increase. Greengard (1990) provides an informal introduction to this.

Carrying out and analysing N-body simulations can be a laborious task.

As in the data reduction phase of an observational project, much time can be saved if the process is sufficiently automated (e.g., Carnevali & Santangelo 1980). It is especially convenient if this can be done within a suitable, special-purpose software environment (Hut & Sussman 1986, Hut et al. 1993).

Hardware advances are also having a big impact. The use of vector machines is now routine, but the application of parallel computers in this problem is still at a rather experimental level, at least for star cluster problems (e.g., Makino & Hut 1989a, Raine et al. 1989, Warren & Salmon 1995, Spurzem 1996). An exception is the code written for a transputer array by Sweatman (1990, 1991, 1993). Use of the Connection Machine is described in Makino & Hut (1989b), Brunet et al. (1990), Theuns & Rathsack (1993) and Hernquist et al. (1995), while Katzenelson (1989) discusses parallelisation of a tree code. The most exciting developments here, however, are in the field of special-purpose hardware, designed and built by a group at Tokyo University under the direction of D. Sugimoto (Sugimoto et al. 1990). These devices fall into two classes. One class (GRAPE-1 and GRAPE-3; cf. Ito et al. 1990) compute forces with relatively low accuracy (Makino et al. 1990, Okumura et al. 1992), but are still suitable for problems which are not dominated by close two-body encounters, binaries, or a high-density core (Hernquist et al. 1992). The other class (GRAPE-2 and GRAPE-4) are not yet quite so widespread, but are ideal for all kinds of problems in star cluster dynamics (Ito et al. 1991, 1993).

One of the great advantages of the N-body technique is that the minimum of simplifying assumptions need be made. By contrast with other methods discussed in later subsections, no assumption is made of spherical symmetry, or isotropy of the velocity distribution. No special steps need be taken to include gravitational interactions involving pairs, triples (e.g., encounters between single stars and binaries), quadruples (e.g. binary-binary interactions), etc. No extra difficulties are created by the inclusion of a spectrum of masses or a tidal field. Indeed, some steps in the direction of greater realism actually make N-body simulations easier. It has even been shown that binaries formed in three-body encounters, which are usually regarded as a bottleneck in these studies, actually become relatively unproblematic when N becomes large enough (Makino & Hut 1990). However, *primordial* binaries will remain time consuming no matter how large N is.

Aside from the scaling with respect to N, discussed below, the main difficulty of the N-body technique is that the results are noisy, because of the relatively small number of stars. The implications of this for the determination of core parameters has been studied in detail by Casertano & Hut (1985), whose work forms the basis for many analyses of N-body results. In fact the statistical noise can be greatly alleviated by averaging results from many simulations (Giersz & Heggie 1993a). More important from the astrophysical point of view, it is very difficult to model rare species (e.g. stellar-mass black holes) with a small N-body simulation. A single massive object may have a different qualitative effect on a small system than the same proportion of objects of the same mass in a large system.

Though not thought to be of practical importance (otherwise it would

undermine the entire N-body modelling effort!), there is in principle one further difficulty in the use of N-body techniques. It stems from Miller's observation (Miller 1964, 1971) that the solution of the N-body equations is highly unstable, on a time scale much smaller than the typical length of a simulation (Kandrup & Smith 1991, 1992; Goodman et al. 1993). Though this means that the positions and velocities of the stars in a simulation are almost certainly quite wrong (Lecar 1968, Hayli 1970b), there is no reason to believe that the statistical results are unreliable provided that the total energy is well conserved (Smith 1977, 1979, Heggie 1991). Numerical "shadowing" results (Quinlan & Tremaine 1991, 1992) provide some reassurance for this point of view. Since energy conservation is the main test available, it is probably wise to avoid techniques which artificially *force* a system to preserve its total energy.

Now we turn to some examples of the requisite scaling of the results. In N-body simulations, it is customary (Hénon 1972, Heggie & Mathieu 1986) to use units in which G, the total mass and the virial radius are unity. Thus one has freedom to choose two of these units, and then the third is determined by the value of G. Equivalently, one has freedom to scale the mass and the unit of time.

The scaling of time depends essentially on the mechanism to be modelled. Phenomena occurring on a crossing time scale (e.g., disk shocking) could be modelled by scaling the crossing time of the N-body model to that of a real cluster. Similarly, modelling of the early evolution, especially the phase in which the cluster adjusts to the rapid loss of mass from the evolution of its massive stars, could be modelled by using the same scaling to determine the stellar evolution time scale within the model. In order to model relaxation effects, including mass segregation, one would scale the mass and half-mass relaxation time to those of an actual cluster. Modelling of the effects of stellar collisions within this context could be added by suitable choice of the stellar radii (McMillan 1993).

Complications arise when the phenomenon to be modelled depends on two or more processes whose time scales scale differently with N. For example, the time scale for formation of a single hard binary is of order Nt_r, and so phenomena involving *both* two-body relaxation and binary formation in a globular cluster cannot easily be modelled with a small N-body simulation. In fact, it was known long ago (Hayli 1970a) that escape due to interactions involving binaries does not scale in the same way as escape due to two-body interactions. Another consequence is that the density of the core at the end of core collapse is N-dependent (Goodman 1987), and this is one reason why gravothermal oscillations (cf. §10.1) have only recently come within reach of N-body models. On the other hand, if primordial binaries are present, then the *formation* of binaries can be neglected, and the evolution of the core can be modelled successfully. (The N-dependence is logarithmic: Heggie & Aarseth 1992).

Another important example is the escape rate, even in the absence of interactions involving binaries. In an isolated system the time scale for escape scales with t_r, except for a logarithmic factor, but in the presence of a tidal

field, especially if it is time dependent (in consequence of the orbital motion of a cluster through its parent galaxy), phenomena which scale as the crossing time are also important. Therefore no straightforward N-body simulation will correctly model these processes.

In view of these difficulties, a number of somewhat modified or ad hoc N-body schemes have been devised. Hybrid schemes (McMillan & Lightman 1984, Aarseth & Bettwieser 1986) approach the problem caused by the differing time scales of binary formation and two-body relaxation by welding a simplified treatment of the latter with an N-body treatment of the central parts (where almost all binaries are formed); nevertheless, they offer only a rather modest advantage of speed (Hut et al. 1988). Similarly, tidal effects on escape have been modelled (Oh et al. 1992) by a simplified treatment of relaxation and careful modelling of the orbit in the tidal potential. Disk shocking has been modelled in a similar simplified way (assuming that relaxation sets up a multi-mass King model between shocks) by Capaccioli et al. (1993). Stability can be studied in an especially economical way by use of suitably modified N-body techniques (Wachlin et al. 1992, Leeuwin et al. 1992), though these last two topics belong to the domain of "collisionless" stellar dynamics, where the scope for short-cuts is much richer.

What is noticeable about these issues is that the use of N-body models in these contexts does not replace the use of theory. Rather, careful consideration of theoretical issues is required before successful simulations can be devised. One of the pitfalls, clearly, may be that there is some slightly subtle and unsuspected interaction between phenomena which scale differently with N. These may remain undiscovered until modelling efforts with the correct values of N become feasible.

In the long run the N-body technique will become the method of choice. So far, however, N-body simulations have not yet been used to model specific clusters, and even their use in the study of open clusters has not made much progress since the work of Terlevich (1985, 1987), except for some remarkable recent developments by Aarseth (1996a,b). One issue that will have to be addressed is how one compares the data from an N-body simulation with observations. An early attempt was von Hoerner's "modulus of evolution" (von Hoerner 1976); i.e., a single parameter whose time dependence is found from N-body simulations and which can be determined observationally (Kadla 1979). This is a test of extreme simplicity, but nothing better has been attempted since then.

8.2 Fokker-Planck methods

One of the fundamental dynamical mechanisms in the evolution of stellar systems is two-body relaxation (§7.1). The theoretical foundations were laid by Chandrasekhar, who introduced a description in terms of a Fokker-Planck equation (Chandrasekhar 1943a,b). His formulation was improved by Rosenbluth

et al. (1957) and the effect of orbital motion was added by Kouzmine (1957). The resulting orbit-averaged Fokker-Planck equation was first put to practical use by Hénon (1961, 1965), whose two papers in this area are long-standing classics. In this formulation the equation resembles the heat conduction equation, and it can be solved numerically by a variety of methods which have all been of importance. They divide into several classes, which we discuss in turn.

First we discuss the Monte Carlo models, which again divide into two types. One was pioneered by Spitzer and his students (Spitzer & Hart 1971a,b, Spitzer & Shapiro 1972, Spitzer & Thuan 1972, Spitzer & Chevalier 1973, Spitzer & Shull 1975a,b, Spitzer & Mathieu 1980), and developed in important ways by Shapiro and his collaborators (Shapiro & Marchant 1978, Marchant & Shapiro 1979,1980, Duncan & Shapiro 1982, Shapiro 1985). The other method was devised by Hénon (1966, 1972, 1973, 1975) and later improved by Stodółkiewicz (1982, 1986). The essential difference in these models is that the former followed the stars around their orbits, and was (in principle) capable of modelling processes occurring on both relaxation and crossing time scales, though in the phenomena actually studied with these models processes of the latter kind were unimportant. Models of Hénon's type, on the other hand, assumed dynamical equilibrium, and that the distribution function depends only on integrals of motion.

Spitzer's method was used to explore a variety of important phenomena, including mass segregation, anisotropy of the velocity distribution, tidal shocking, the role of primordial binary stars, etc., and the above sequence of papers is often a good starting point for information on these areas. At first, Hénon's method was used to explore somewhat more idealised problems – for example it was the first to break through the impasse of core collapse (Hénon 1975), but it was brought to an amazing level of realism by Stodółkiewicz (1984, 1985). Indeed from this point of view his papers remain unsurpassed: they included such processes as the formation of binaries by two- and three-body encounters, mass loss from stellar evolution, tidal shocking, etc.

In view of the success of the Monte Carlo method it is surprising that it has been ignored in the last few years. One reason is that it faced vigorous competition from a direct numerical (finite-difference) solution of the Fokker-Planck equation, along lines pioneered by Cohn (1979, 1980). Similar methods had been developed for a fixed potential by Ipser (1977) and by Cohn & Kulsrud (1978), and since then codes like Cohn's have been written independently by Inagaki & Wiyanto (1984) and by Chernoff & Weinberg (1990). Like the Monte Carlo methods, Cohn's formulation assumes spherical symmetry, though codes which can handle a rotating cluster have been devised by Goodman (1983a) and by Einsel & Spurzem (1996). More importantly, it is usual to assume that the distribution of velocities is isotropic, which was not customary in the Monte Carlo models. One of the main reasons for this simplification is that there exists in this case a numerically very well behaved scheme due to Chang & Cooper (1970). Over the years there have been several unsuccessful attempts to develop something comparable for anisotropic models, whose numerical behavior was therefore less satisfactory as judged by energy conservation (Cohn

1985). Recently, however, Takahashi (1995, 1996) has demonstrated a new approach, which definitely appears to have cured finite difference methods of this long-standing problem.

It is still true that the introduction of anisotropy greatly increases the computational effort, as it resembles the transfer from one-dimensional to two-dimensional diffusion. Similarly, introduction of unequal masses greatly increases the time taken to compute a model. Equally demanding computationally is the introduction of further physical processes, e.g., binaries, whether those formed by two body encounters (Statler et al. 1987, Lee 1992, Lee & Ostriker 1993) or those present initially (Gao et al. 1991). In the latter case, for example, it was necessary to assume that the distribution of the energies of the binaries was independent of their spatial distribution, whereas N-body models of different kinds show that more energetic binaries are found at larger radii (McMillan et al. 1990, Hut et al. 1992b, Heggie & Aarseth 1992). Without the inclusion of such processes it is unlikely that any satisfactory models for the advanced evolution of globular clusters can be constructed (cf. Drukier et al. 1992). For all these reasons the most realistic Fokker-Planck models, like the largest N-body models, require large-scale computing facilities.

A third technique for solving the Fokker-Planck equation has been under development for a number of years now. Based on the variational formulation of Inagaki & Lynden-Bell (1990), it now appears to be roughly competitive with finite difference methods. To begin with it was developed and used successfully for isotropic models (Takahashi & Inagaki 1992; Takahashi 1992, 1993). Recently, however, it has been rapidly developed to the point where accurate anisotropic models can be used to follow the evolution through core collapse into the post-collapse regime (Takahashi 1995, 1996).

It is also worth mentioning a fourth formulation, in terms of path-integrals (Horwitz & Dagan 1988, Dagan & Horwitz 1988), which yet again offers different (and unexplored) possibilities for numerical work. Finally, Luciani & Pellat (1987b) have presented a form of the Fokker-Planck equation which makes minimal assumptions about symmetry and the distribution function, though it has not yet been put to practical use.

A final question mark over Fokker-Planck models is their validity. Work by Hénon (1975) revised the theory of the relaxation time which had been used in the classic paper of Aarseth et al. (1974), and found that the results of Fokker-Planck and N-body models were brought into satisfactory agreement. This refers to the case of isolated models with equal or unequal masses, and recent work by Giersz & Heggie (1993a, 1994, 1996a) and Giersz & Spurzem (1994; cf. Fig. 7.1 above) has strengthened and refined Hénon's conclusion. For more realistic models the situation is less satisfactory. The formula that is often used for the rate of energy generation by binaries in multi-component models (e.g., Lee et al. 1991) rests on a very slender foundation. The Fokker-Planck treatment of tidal effects is necessarily spherically symmetric and often simplified to imposition of a tidal cutoff (e.g., Chernoff & Weinberg 1990), though in this case a somewhat more realistic formulation has been devised (Lee & Ostriker 1987). Fukushige & Heggie (1995) find that the lifetime of

N-body models in the face of tidal disruption can greatly exceed that found using a Fokker-Planck model. In this case it is the assumption of dynamical equilibrium which seems to be at fault.

Fig. 8.2. Main sequence mass function index as a function of projected radius (in pc), at various times during the evolution of a Fokker-Planck model (from Chernoff & Weinberg 1990 Fig. 35). The index is defined by $dN \propto m^{-\alpha} dm$. Times are given in years. Initial conditions are a King model with $W_0 = 7$, $\alpha = 2.5$ for 0.4 $M_\odot <$ m $<$ 15 $M_\odot$, total mass 2.82×10^5 $M_\odot$, at galactocentric distance 10kpc.

Despite these shortcomings, the direct Fokker-Planck method is at present the most important and widely used source of information on a wide range of essential phenomena, including mass segregation (Inagaki & Wiyanto 1984), and the additional effects of tidally-induced escape and mass loss from stellar evolution (Weinberg & Chernoff 1989, Chernoff & Weinberg 1990). Their 1990 paper, with an update in Chernoff (1993), is another excellent starting point for learning what the Fokker-Planck model can teach us about the evolution of model star clusters (cf. Figs. 7.3 and 8.2).

As was mentioned at the beginning of this section, the Fokker-Planck equation treats relaxation as a diffusion process. It can also be treated in a manner more closely resembling the Boltzmann equation, i.e., by a formulation in which the distribution function evolves by discrete changes in the energies of the stars. The equation to be solved is then an integral-differential equation (Petrovskaya 1969a,b, 1971; Kaliberda & Petrovskaya 1970). Appropriate

numerical techniques have been devised (Goodman 1983b), but are much less well developed than for the Fokker-Planck equation (see §7.1 for references to the basic theory).

8.3 Conducting gas models

The resemblance between a star consisting of huge numbers of atoms and a star cluster or galaxy containing large numbers of stars becomes clear at quite a simple level, e.g., the virial theorem. It is more surprising, but still true, that the resemblance extends to phenomena such as heat transport, energy generation, and core-halo evolution. For the study of the dynamical evolution of stellar systems this resemblance was first exploited by Larson (1970a), whose work was the first to exhibit the time-dependence of core collapse.

Models of this kind can be divided into two classes. Larson's, which was taken up by Angelleti & Giannone in an important but unjustly neglected series of papers (Angeletti & Giannone 1976, 1977a,b, 1978, 1979, Angeletti et al. 1980), and has been developed further recently by Louis & Spurzem (1991), should really be regarded as an approximate solution of the Fokker-Planck equation, obtained by studying the moments of the velocity distribution. Since these are essentially the density, the mean velocity of the stars, the (kinetic) energy density, etc., the resulting equations resemble those of stellar evolution. The other method, somewhat more phenomenological, starts with the equations of stellar evolution and corrects the physics: no radiative energy transport, and conduction altered to suit the effects of two-body relaxation. Except for the last point (Lynden-Bell & Eggleton 1980) the method was introduced by Hachisu et al. (1978), and it was subsequently developed and exploited by Bettwieser (1983, 1985a,b) and others.

The first remarkable point about these models is that they work, i.e., they give results which closely resemble those of Fokker-Planck and N-body models in many respects (Aarseth et al. 1974, Bettwieser & Sugimoto 1985, Giersz & Heggie 1993a, 1994, 1996a, cf. Fig. 7.1 above). It is not obvious why this should be so, because the treatment of conduction is so different: at each radius it is governed by local conditions, whereas the orbital motion of stars implies that encounters at one radius can and should affect the distribution functions at widely different radii. One phenomenon where this seems to be important is in the growth of anisotropy (Giersz & Spurzem 1994). A speculative reason for the success of the gas model is that so many aspects of the evolution of stellar systems have a thermodynamic basis, and this is accurately described in these simple models.

The foregoing remarks show that the gas model of stellar systems has been developed to include a variety of phenomena, though not quite to the same extent as the Fokker-Planck model (cf. Table 7.1 above). A spectrum of stellar masses can be included (Heggie & Aarseth 1992, Spurzem & Takahashi 1995), despite unsatisfactory results of an earlier attempt (Bettwieser & Inagaki

1985). Other developments include simple treatments of the effects of stellar mass loss (Angeletti & Giannone 1977c, 1980), binary formation and evolution (Heggie 1984, Heggie & Ramamani 1989), even including stochastic (random) effects (Giersz & Heggie 1994) or primordial binaries (Heggie & Aarseth 1992). At one time the gas model led the field in producing a dynamical evolutionary model of a specific cluster in which the photometric properties of different kinds of star were included for the production of multi-color surface density profiles (Angeletti et al. 1980).

In general it may be said that the gas model has the same merits and demerits as the Fokker-Planck model, except for two points: its advantage is that it is quicker, but each new problem must be approached with caution, as it is not really clear why it works as well as it does, and its treatment of relaxation is inferior. Its main use is as a quick tool for exploring an area which can be followed up later by better methods.

Most of the N-body, Fokker-Planck, and conducting gas studies have been theoretically oriented, having in mind the numerical investigation of questions linked to the dynamical evolution. Their results are presented in §9. Only a few Fokker-Planck models have been directly fitted to observations. Recent exceptions include the work of Grabhorn et al. (1992) on M15 and NGC 6624, that of Phinney (1993), also on M15, and especially the detailed study of NGC 6397 by Drukier (1993, 1995, and cf. §9.2 below).

9. Towards catastrophic phases of evolution ?

In the present section we resume our discussion of dynamical evolution. We have already discussed the early evolution governed by processes on the time scale of the crossing time and that of mass loss from the evolution of massive stars (cf. §5). Assuming that the cluster has then settled into a state of quasi-static dynamical equilibrium, we discussed suitable models in §7.5. Now we turn to the effects of two-body relaxation on time scales of several relaxation times, i.e., what is sometimes called "secular evolution". Another way of expressing the position is that we now turn to evolution on a "thermal" time scale (cf. the discussion of the gas model in §8.3), whereas parts of §5 concern processes acting on a dynamical time scale. Many of the phenomena we discuss can only be modelled adequately using the techniques of the previous section, but now we concentrate on the results.

9.1 The gravothermal instability and core collapse

For many years (between about 1940 and 1960) secular evolution was understood in terms of the "evaporative model" of Ambartsumian (1938) and Spitzer (1940). In this model it is assumed that two-body relaxation attempts to set

up a maxwellian distribution of velocities on the time scale of a relaxation time, but that stars with velocities above the escape velocity promptly escape. If the escaping stars carry off little energy, this model predicts that the number of stars in the cluster varies approximately as $(t_0 - t)^{2/7}$, where t_0 is a constant representing the time at which the entire cluster will have evaporated. The evolution of its size, velocity dispersion, etc., can be estimated equally simply. For example the density varies as:

$$\rho \propto (t_0 - t)^{-10/7}. \tag{9.1}$$

The next major step in understanding came when it was discovered that evolution arises also when stars escape from the inner parts of the cluster to larger radii, without necessarily escaping altogether. Antonov (1962) realised that these internal readjustments need not lead to a structure in thermal equilibrium, because thermal equilibrium may be unstable in self-gravitating systems. The explanation of Lynden-Bell (1968) is worth repeating. Consider a conducting, self-gravitating gas enclosed by a spherical wall. (In a real system the inner parts are kept in by the pressure of the outer layers, but this does not change anything qualitatively.) If the core is warmer than the part inside the wall, thermal energy flows outwards. The outer region, which is held in by the wall, heats up. But the inner part also heats up because it is pressure-supported: loss of thermal energy reduces the pressure, and in the subsequent slight collapse gravitational potential energy is converted into kinetic energy. Whether the temperature difference between the inner and outer parts is greater than it was before depends, among other things, on the heat capacity, and therefore the mass, of the outer layer. If it is sufficiently great (i.e., the core is sufficiently compact), the initial temperature excess of the core is enhanced, and with it the conduction of heat and the collapse of the core.

In view of the central part played by Antonov's instability, it has been investigated from various points of view. Lynden-Bell & Wood (1968) reworked Antonov's theory, related it to other stability criteria (depending on the boundary conditions), and followed up some consequences for the evolution of stellar systems. They also carried out a similar analysis for the King and Woolley sequences of models, as well as for the isothermal case. Though Taff & van Horn (1975) claimed that this analysis was faulty, similar results were obtained (by different techniques) in a series of papers by Horwitz & Katz (1977, 1978), Katz & Horwitz (1977) and Katz et al. (1978); see also Ipser (1974), Larson (1970a), Katz (1978, 1980) and Padmanabhan (1989a, 1990) for yet other approaches. A particularly readable account of the thermodynamic basis of the instability was provided by Hachisu & Sugimoto (1978) in the context of gaseous models. Nakada (1978), Inagaki (1980) and Luciani & Pellat (1987a) provided stability analyses on the basis of the conducting gas model, the isotropic Fokker-Planck equation, and the anisotropic Fokker-Planck equation, respectively. The role of boundary conditions in the stability of N-body systems was explored by Miller (1973), and the stability of (singular) anisotropic models was investigated by Spurzem (1991).

The "gravothermal instability", as it is often called, is considerably complicated in the case of unequal masses. A classical paper by Spitzer (1969) showed that the scope for thermal equilibria (which requires equipartition of energies) is even more restricted than in the case of equal masses (see also Lightman 1977, Yoshizawa et al. 1978, Katz & Taff 1983). Addition of rotation adds further features (Inagaki & Hachisu 1978, Tajima 1981, Lagoute & Longaretti 1996, Longaretti & Lagoute 1996a,b).

What the above discussion does not make clear is the dynamical consequence of the instability. The well known process of "core collapse" is usually interpreted as a manifestation of the gravothermal instability. It is best studied using the techniques discussed in §8, but various simplified and more or less instructive models have been devised to illustrate the link between the two. Several authors (Da Costa & Pryor 1979, Da Costa & Lightman 1979, Danilov 1989) have constructed such models of the evolution of core and halo in terms of the energy and mass exchanged between them. Already Lightman & Fall (1977) had provided a theory along similar lines for two-component systems. It turns out that the way in which the relaxation time depends on density and velocity dispersion is crucial to the way the instability develops (Makino & Hut 1991).

Now we summarise some results of more detailed models, mostly made using the Fokker-Planck method (cf. also Spitzer 1984, 1985 for other summaries). Even in systems of stars with equal mass, and assuming an isotropic distribution of velocities, the time-dependence is a little complicated. Expressed in terms of t_{rc}, the relaxation time in the core, the e-folding time for the evolution of the central density varies from about 5 in the early stages (assumed here to be a Plummer model) to about 330 in late phases (Cohn 1980). The time scale for the evolution of the velocity dispersion is generally much longer, as this quantity varies much less than the central density. However, the increase of the projected central velocity dispersion is sufficient to show up in even quite small N-body simulations (Struble 1979). The time scales in late core collapse are somewhat longer if the distribution of velocities is allowed to be anisotropic (Takahashi 1995, who obtained a result contrary to that of Cohn 1979).

Late in the process of core collapse the evolution of the central density, velocity dispersion, etc., becomes simple, and resembles that in the evaporative model (Eq. 9.1), but with somewhat different indices. The reason for this is that the deep evolution is driven by interactions within the inner parts of the system, and so the influence of the outer parts of the cluster become negligible (Lynden-Bell 1975). The corresponding self-similar evolution was revealed first by Lynden-Bell & Eggleton (1980) using an isotropic gas model, then by Heggie & Stevenson (1988) with the Fokker-Planck model, and finally by Louis (1990), who used an anisotropic gas code.

For an isolated cluster (without a tidal field) the time scale for the entire evolution of the core (when the density has formally become infinite) is about 15.7 $t_{rh}(0)$, when expressed in terms of the initial half-mass relaxation time (Cohn 1980). This result is for an isotropic code starting from a Plummer

model with stars of equal mass, and for an anisotropic code the time extends to 17.6 $t_{rh}(0)$ (Takahashi 1995). Results for collapse from a King model, with or without a steady tide, are given by Wiyanto et al.(1985), Inagaki (1986a) and Quinlan (1996).

Fig. 9.1. Evolution of Lagrangian radii in an N-body model with a mass spectrum, a steady tide and mass loss from stellar evolution (from Aarseth & Heggie, in preparation). Initial conditions: $dN \propto m^{-2.5}dm$, King model with $W_0 = 7$, mass $1.5 \times 10^5\ M_\odot$, galactocentric radius 4 kpc, orbital speed 220 $\mathrm{km\,s^{-1}}$. Time is in Myr, the unit of length is 6 pc. Though the model has $N = 8192$ stars initially, results are scaled to the above initial conditions. The radii plotted are (from the top) tidal radius (r_t), and Lagrangian radii corresponding to fractions 0.5, 0.1, 0.01 and 0.001 of the mass inside r_t. The initial rise takes place in the early phase of intensive mass loss by stellar evolution. Core collapse is complete at about 10 Gyr, which compares very well with the value 9.6 Gyr obtained by Chernoff & Weinberg (1990) using Fokker-Planck techniques. The evolution around core bounce and beyond would probably be significantly altered if an appropriate population of primordial binaries had been included. Tidal shocks were not included.

The collapse time is generally shorter in the presence of unequal masses (Inagaki & Wiyanto 1984, Chernoff & Weinberg 1990). For example, a tidally limited cluster starting from a King model with scaled central potential $W_0 = 3$, with stellar masses having a power-law distribution with index $x = 2.5$ over a range from 0.4 to 15 $M_\odot$, takes about 0.9 $t_{rh}(0)$ for complete collapse (cf.

Fig. 7.3). Up to a point this reduction can simply be understood because the equipartition time scale of the most massive stars varies inversely with mass (Eq. 7.6). At any rate, results such as these contradict the conclusion of Sygnet et al. (1984), who asserted that stellar systems cannot have suffered a gravothermal catastrophe because the time scale is too great.

Murphy & Cohn (1988) give surface brightness and velocity dispersion profiles at various times during collapse, for a system with a reasonably realistic present-day mass spectrum. Addition of effects of stellar evolution, modelled as instantaneous mass loss at the end of main sequence evolution, delays the onset of core collapse (Angeletti & Giannone 1977c, 1980, Applegate 1986, Chernoff & Weinberg 1990, Kim et al. 1992).

As already mentioned, most of the foregoing results stem from Fokker-Planck studies. Goodman (1983b) has shown that late core collapse proceeds in much the same way if a better model is used (which does not make the same assumption of small-angle scattering). Examples of N-body models which illustrate various aspects of core collapse include Aarseth (1988), where $N = 1,000$, Giersz & Heggie (1993a) ($N \leq 2,000$), Spurzem & Aarseth (1996) ($N = 10,000$), and Makino (1996a,b; see Fig. 10.1 below) ($N \leq 32,000$) and Fig. 9.1.

The effect of a *time-dependent* tidal field can be to accelerate core collapse (Spitzer & Chevalier 1973). This may be a significant point for the interpretation of observations (cf. §§9.2, 9.3, and 9.8), which show that galactic globular clusters with collapsed cores are concentrated towards the galactic centre, where disk shocking is indeed stronger and more frequent. Note, however, that the mean density of tidally limited clusters is higher towards the galactic centre, and so the collapse time for clusters of a given mass would be shorter near the galactic centre even in a steady tidal field.

Although the mass of the core decreases during core collapse, the inner parts of the cluster do flow inwards throughout most of the collapse phase, and it can be understood from energy conservation that the outer parts of the cluster move outwards, unless the cluster is limited by the tidal field of the galaxy. The half-mass radius is relatively fairly static (Figs. 7.1 and 9.1).

9.2 Observational evidence of core collapse through the density profile and concentration

In the eighties, CCD observations allowed a systematic investigation of the inner surface brightness profiles (within $\sim 3'$) of 127 galactic globular clusters (Djorgovski & King 1986, Chernoff & Djorgovski 1989, Trager et al. 1995). These authors sorted the globular clusters into two different classes illustrated in Fig. 9.2: (i) the "King model clusters", whose surface brightness profiles resemble a single-component King model with a flat isothermal core and a steep envelope, and (ii) the "collapsed-core clusters", whose surface brightness profiles follow an almost pure power law with an exponent of about -1. In the Galaxy, about 20% of the globular clusters belong to the second type,

exhibiting in their inner regions apparent departures from King-model profiles. Consequently, they are considered to have collapsed cores.

Similar independent surveys in the Magellanic Clouds (Meylan & Djorgovski 1987, Mateo 1987) show possible signs of a collapsed core in three high-concentration old clusters in the Large Magellanic Cloud, viz., NGC 1916, NGC 2005, and NGC 2019, all three of which are projected on the bar close to the centre of the LMC. No such cluster is observed in the Small Magellanic Cloud. Bendinelli et al. (1993) and Fusi Pecci et al. (1994) announce the first detection, thanks to the high spatial resolution of HST, of a collapsed-core globular cluster in M31, viz., G 105 $\equiv$ Bo 343. See also Grillmair et al. (1996) and Jablonka et al. (1996) for other clusters in M31..

For quite a few of the 125 globular clusters for which CCD observations of their cores have been obtained (Trager et al. 1995), within $\sim 3'$ from the centre, there exists also aperture photometry at intermediate radii, and star counts at large radii which allow the construction of surface brightness profiles extending from the core out to 30-50$'$. Nevertheless, discrimination between the two different classes — the "King model clusters" and the "collapsed-core clusters" — may often be less clear-cut than it might seem from Fig. 9.2. There are two main reasons for this:

Fig. 9.2. Examples of surface brightness profiles (Djorgovski et al. 1986 Fig. 1). Left: NGC 6388 resembles a King-model with a flat isothermal core and a steep envelope. Right: Terzan 2 is an example of a collapsed-core cluster whose light profile follows an almost pure power law with an exponent of –1.

(i) Statistical noise. Integrated surface brightnesses measured for small areas in the cores of globular clusters are strongly dominated by statistical fluctuations in the small numbers of bright stars within the aperture. Such fluctuations are conspicuous, e.g., in the case of NGC 6397, whose ground-

based surface brightness profile, in its inner 100″, increases toward the centre in a very bumpy way, especially when observed through B or U filters, because of a high central concentration of bright blue stragglers (Djorgovski & King 1986, Aurière et al. 1990, Meylan & Mayor 1991, Drukier 1995). The power-law shape of the inner observable part of a high-concentration profile may be difficult to observe because of statistical fluctuations in the small numbers of bright stars (Sams 1995). One way to alleviate this problem consists of using HST data, allowing star counts (King et al. 1995).

(ii) Similarity between high-concentration King models and power-law profiles. The inner parts of King models of very high concentrations (cf. King 1966 Fig. 1) have profiles which resemble that of a singular isothermal sphere. See also §7.2. Consequently, high-concentration King profiles can be successfully adjusted to the surface brightness profiles of so-called "collapsed-core clusters", as illustrated in Meylan (1994 Fig. 2). For example, multi-mass King-Michie models fit the surface brightness profile of NGC 6397 reasonably well and have very high concentrations, viz., $c = \log{(r_t/r_c)} \simeq 2.5$ (Meylan & Mayor 1991, Drukier 1995, King et al. 1995). In a similar way, but see also Illingworth & King (1977) and King (1989), Grabhorn et al. (1992) are able to fit successfully a multi-mass King model of even higher concentration, viz., $c = \log{(r_t/r_c)} \simeq 3.0$, to the surface brightness profile of M15, the prototype of the collapsed-core globular clusters. There is no evidence that fits of King models to post-collapse clusters are any less satisfactory than those to uncollapsed clusters.

This indicates that the general dynamical status (is it collapsed or not?) of a cluster may be straightforwardly deduced from the value of its concentration, without careful study of the power-law shape of its surface brightness profile. Consequently, any globular cluster with a concentration $c = \log{(r_t/r_c)} \gtrsim$ 2.0-2.5 may be considered as collapsed, or on the verge of collapsing, or just beyond. It is worth mentioning that the pre-, in-, and post-collapse terminology encountered in the literature has only a theoretical meaning, since (so far) observations are unable to differentiate these three different phases. The dynamical status evaluated from the concentration value also has the advantage of alleviating the outstanding problem implied by the many clusters — like 47 Tucanae — which show no trace of "collapsed-core cluster" morphology, but have short enough dynamical times to have collapsed in a small fraction of the Hubble time.

Although ground-based data are essential for most globular clusters, any study aiming at resolving the core of the densest galactic globular clusters is possible only with the refurbished HST. For example, detailed photometric studies have been published on the core of NGC 6397 by King et al. (1995), of NGC 6624 by Sosin & King (1995), of NGC 6752 by Shara et al. (1995), and of M30 by Yanny et al. (1994b). Although most clusters have a resolved core, a few clusters — e.g. M15 and NGC 6624 — show inner star-count profiles which do not suggest any sign of levelling off.

As already briefly mentioned in §6.1, the globular cluster M15 has long been considered as a prototype of the collapsed-core star clusters. Early elec-

tronographic determinations of its luminosity profile by Newell & O'Neil (1978), confirmed by further photographic and CCD studies (e.g., Aurière & Cordoni 1981a,b), reveal a central excess of light. Newell, Da Costa, & Norris (1976) found that these observations were consistent with the existence of a central massive object, possibly a black hole of about 800 $M_\odot$, while Illingworth & King (1977) were able to successfully fit dynamical models to the entire surface-brightness profile, invoking a centrally-concentrated population of neutron stars instead of a black hole.

Fig. 9.3. A $5.6'' \times 3.5''$ part of an FOC image of the centre of M15 (from King 1996 Fig. 1). The pixel size is $0.014''$. The white areas are FOC saturation in the brightest stars of this field. The three bright stars — AC 214, 215, and 216 — forming a near equilateral triangle near the centre of the image are the main contributors to the former bright luminosity cusp. But see Fig. 9.4 for the faint stars radial density profile.

Now, high-resolution imaging of the centre of M15 has resolved the luminosity cusp into essentially three bright stars (§6.1 and Aurière et al. 1984, Racine & McClure 1989, Lauer et al. 1991, Yanny et al. 1993, 1994a, and Sosin & King 1996). On the one hand, Lauer et al. (1991) show that the surface-brightness profile of the residual light, obtained after subtracting the bright resolved stars, does not continue to rise at subarcsecond radii. They determine a core radius of $2.2'' = 0.13$ pc. On the other hand, also from pre-refurbishment HST data, Yanny (1993) and Yanny et al. (1993, 1994a) find that a flat core is not apparent for $r \gtrsim 1.5''$. They find the radial distribution consistent with a number of scenarios, including: i) a central black hole of mass a few times 10^3

$M_\odot$; *ii*) a collapsed core with steep central profile of slope < -0.75, and *iii*) a small flat core of radius $\lesssim 1.5'' = 0.09$ pc. Earlier reports of a weak color gradient in the centre of M15 (Bailyn et al. 1988) are confirmed in the sense that bright red giants are depleted in the centre relative to subgiants, but the depletion of very blue horizontal-branch stars counteracts this bluing (Stetson 1991, De Marchi & Paresce 1994b).

In such a subtle matter, for which data processing methods are not free of influence, star counts are far superior in quality to any surface brightness measurement. Post-refurbishment HST star-count data confirm that the $2.2''$ core radius observed by Lauer et al. (1991), and questioned by Yanny et al. (1994a), is observed neither by Guhathakurta et al. (1996b) with WFPC2 data nor by Sosin & King (1996) with FOC data.

A $5.6'' \times 3.5''$ area of an FOC image centered on the core of M15 is displayed in Fig. 9.3. The completeness-corrected surface-density profile of stars with V magnitude between 18.5 (just above the main-sequence turn-off) and 20.0 is shown in Fig. 9.4 from Sosin & King (1996). All the 839 stars have nearly the same mass. This surface-density profile clearly continues to climb steadily within $2''$. A maximum-likelihood method rules out a $2''$ core at the 95% confidence level. It is not possible to distinguish at present between a pure power-law profile and a very small core.

Fig. 9.4. The completeness-corrected surface-density profile of stars with V magnitude between 18.5 and 20.0 (from Sosin & King 1996 Fig. 1). The value of $2.2''$ obtained by Lauer et al. (1991) for the core radius is illustrated by the vertical dotted line.

Density profiles of M15, very similar to those of Sosin & King (1996), deduced from star counts obtained with WFPC2 data are given by Guhathakurta et al. (1996b) for two independent magnitude-limited samples: $V < 18.3$ and

$18.3 < V < 20.0$. These two radial profiles are the same, within Poisson errors. This is to be expected, since the difference between the average masses of the stars ($\sim 0.75\ M_\odot$ for $V = 18.3$-20.0 and $\sim 0.78\ M_\odot$ for $V < 18.3$) is expected to be too small to give rise to significant effects due to mass segregation. Guhathakurta et al. (1996b) provide three different approaches to measuring the surface density distribution: binned star counts, parametric fits, and non-parametric estimates. The density profile appears to rise smoothly towards the centre of the cluster, with no suggestion of levelling off. It can be approximated, in the range from $0.3''$ to $6''$, by a power law r^α with $\alpha = -0.82 \pm 0.12$. This value is similar to that expected for the stellar distribution around a black hole with a mass of a few times $10^3\ M_\odot$ (Bahcall et al. 1975, Bahcall & Wolf 1976, 1977) although it is fully consistent with core-collapse models, which offer a somewhat simpler, less exotic, explanation (see, e.g., Murphy & Cohn 1988, Murphy et al. 1990, Grabhorn et al. 1992; see also Goodman 1993 and references therein).

The most recent, best quality HST data show no evidence of any levelling off of density profiles in the cores of the most concentrated globular clusters like M15 and M30. This can be interpreted as a direct evidence of core collapse, from the density profile. The best current studies are limited by the uncertainties in the cluster centroid position, in the correction factor for crowding problems in star counts, and by Poisson error in the counts, which restrict the analysis of the surface density profile to $r > 0.3''$.

9.3 Observational evidence of core collapse through the velocity dispersion profile

Contrary to the spatial resolution of imaging techniques, which has improved by about an order of magnitude (from $\sim 1.0''$ to $\sim 0.1''$) with the advent of HST, the spatial resolution of spectroscopic capabilities applied to the measurement of the velocity dispersion in globular clusters to about 1 $\mathrm{km\,s^{-1}}$ is still of the order of $\sim 1.0''$. Consequently, the search for a cusp in velocity dispersion profiles is far less advanced than the search for cusps in density profiles. This is really a pity, since the definitive way to distinguish between core-collapse and black-hole models consists of measuring, as a function of the radius, the dispersion of the radial velocities of as many stars as possible in the central regions.

Once again, M15 looks like the most interesting candidate, being the only globular cluster which has been studied carefully for the presence of a central cusp in velocity dispersion. Cudworth's (1976) proper motion study gave the first estimate of velocity dispersion in M15, $\sigma_p \sim 10\ \mathrm{km\,s^{-1}}$, based on stars between $1.5'$ and $12'$ from the centre. Peterson et al. (1989) published the first velocity dispersion profile in M15, derived from two different kinds of data: (i) from individual radial velocities for 120 cluster members scattered between $0.1'$ and $4.6'$ from the centre and (ii) from integrated-light spectra of the central

luminosity cusp. The radial velocities of the 27 stars within 20″ of the centre give $\sigma_p = 14.2 \pm 1.9$ km s^{-1}, while the integrated-light spectra suggest a cusp in the velocity dispersion profile, with $\sigma_p(0)$ of at least 25 km s^{-1}, a unique case among globular clusters. This central value does not match the predicted velocity dispersion profile from Fokker-Planck models (Grabhorn et al. 1992).

Fig. 9.5. Upper panel: superposition of two images of the central 10.5″ × 10.5″ region of M15. The contour plot comes from a V-band image of angular resolution of 0.35″ taken with HRCam at the CFHT (Racine & McClure 1989 Fig. 1). The black dots are stars from an HST FOC image obtained with the F342W filter and displayed with a high low-cutoff so as to show only the sharp cores (0.08″ FWHM) of the bright star. The near equilateral triangle, mentioned in Fig. 9.3, formed by three bright stars near the centre of the image is easily recognisable. The five dashed-line rectangles show the different positions of the 1″ × 8″ slit during five high-resolution spectroscopic observations of the cluster core by Dubath & Meylan (1994). For the purpose of illustration of the simulations by Dubath et al. (1994), an integrated-light spectrum has been extracted from each of the three particular areas indicated by three solid-line rectangles and labelled I, II, and III. Lower panel: the three cross-correlation functions corresponding to the three integrated-light spectra from areas I, II, and III, respectively. From Dubath & Meylan (1994 Fig. 2).

As part of a long-term program to determine the central velocity dispersion in the cores of high-concentration and collapsed-core globular clusters, Dubath et al. (1993a,b, 1994) obtained an integrated-light spectrum of the core

of M15, over a central $6'' \times 6''$ area, leading (see Eq. 6.1 above) to a projected velocity dispersion $\sigma_p(0) = 14.0$ km s^{-1}. It is worth mentioning that, because of a larger sampling area, Dubath et al. (1994) would have probably missed any central cusp in velocity dispersion.

Totally unexpectedly, and despite the high signal-to-noise ratio of the observed spectrum, the cross-correlation function of the M15 spectrum is bumpy, as if it were the sum of two different gaussians. This large departure from the usual gaussian function (see Fig. 6.1 and Fig. 6.2 above) is larger than the deviations produced by the spectrum noise. Such a behavior (also present in the cross-correlation function displayed in Fig. 10 of Peterson et al. 1989) is expected only if the integrated-light spectrum is completely dominated by the contribution of the few brightest stars lying inside the sampling area (slit) of the spectrograph.

A quantitative estimate of the small number statistics from a few bright stars affecting the central velocity dispersion measurements of M15 is absolutely necessary for further interpretations of any results. Detailed and exhaustive numerical simulations, with different sampling apertures ($1'' \times 1''$ in the case of Peterson et al., and $6'' \times 6''$ in the case of Dubath et al.), of the cross-correlation functions of integrated-light spectra in the core of M15 have been carried out by Dubath et al. (1993, 1994). See also Zaggia et al. (1992a,b, 1993) for similar simulations, (originally related to their observation of another globular cluster, viz. M30, but adapted to Peterson et al.'s observations of M15).

The results of these simulations may be summarized by two points: (i) when the light in the sampling area is dominated by one bright star, the observed cross-correlation function is narrow, similar to that of a standard star, and the derived velocity dispersion is too small (see Fig. 9.5, area I, in lower-left panel); (ii) when the light in the sampling area is dominated by two bright stars with unusually different radial velocities, the observed cross-correlation function is broadened because of its double dip, and the derived velocity dispersion is too large (see Fig. 9.5, area III, in lower-right panel). The noisy shapes of Peterson et al.'s and Dubath et al.'s observed cross-correlation functions of M15 are qualitatively easily reproduced by the simulations. They show that any velocity dispersion obtained from integrated-light measurements over small central areas suffers from large statistical errors due to the small numbers of bright stars present in the integration area.

Two complementary observational studies have given a partial (maybe not final) solution to the conundrum presented by the velocity dispersion in the core of M15.

First, Gebhardt et al. (1994, 1995) have used the Rutgers Imaging Fabry-Perot Spectrophotometer to measure radial velocities with uncertainties of less than 5 km s^{-1} for 216 stars within $1.5'$ of the centre of M15, with a technique which can alleviate the problems due to crowding and sampling. Their velocity dispersion profile is plotted in Fig. 9.6. The small dots are the absolute values of each star's deviation from the cluster velocity plotted versus the distance from the centre. The solid line and the open circles, with error estimates, are the velocity dispersion estimated by two different techniques: (i) the open

circles are the maximum likelihood estimate of the dispersion in bins of 22 stars; (ii) the solid line is a locally weighted scatterplot smoothing fit to the velocity deviations squared. The velocity dispersion profile shows a sharp rise from 7 to 12 $\mathrm{km\,s^{-1}}$ at $0.6'$ (1.8 pc) and then appears to flatten into the innermost point at $0.15'$. The dispersion at $30''$ obtained by Gebhardt et al. (1994) is 10 $\mathrm{km\,s^{-1}}$, and it reaches its maximum, of only 12 $\mathrm{km\,s^{-1}}$, at $20''$. At smaller radii the dispersion remains constant. Their data confirm the rise in velocity dispersion seen by Peterson et al. (1989) between $0.7'$ and $0.4'$, but give a velocity dispersion estimate about 1.7 standard deviations lower in the region between $0.1'$ and $0.4'$. Because of their $1.8''$ seeing Gebhardt et al. (1994) cannot determine the dispersion accurately within the central few seconds of arc.

Fig. 9.6. Velocity dispersion as a function of the radius, for stars in M15 (from Gebhardt et al. 1994 Fig. 5). The dots are the absolute deviations from the cluster velocity of the individual radial velocity measurements. The open circles are the velocity dispersion estimates, with uncertainties, in bins of 22 stars. The solid line is a locally weighted scatterplot smoothing fit to the velocity deviations squared and the dashed lines are its 90% confidence interval. The central determination of the velocity dispersion $\sigma_p = 11.7 \pm 2.6$ $\mathrm{km\,s^{-1}}$ (Dubath & Meylan 1994) is represented by the large filled circle.

Second, using the ESO New Technology Telescope, Dubath & Meylan (1994) have obtained five high-resolution integrated-light echelle spectra over the core of the M15. They used a $1'' \times 8''$ slit, with a $1''$ offset between each exposure in order to cover a total central area of $5'' \times 8''$ (Fig. 9.5, upper panel). By taking advantage of the spatial resolution along the slit, they extracted spectra at 120 different locations over apertures $\sim 1''$ square. The Doppler

velocity broadening of the cross-correlation functions of these integrated-light spectra is always ≤ 17 km s^{-1} ($6.5 \leq \sigma_p \leq 17.0$ km s^{-1}), at all locations in the $5'' \times 8''$ area. These observations confirm that the cross-correlation functions of integrated-light spectra taken over such small apertures are mostly dominated by the contribution of one or two bright stars, leading to unreliable estimates of the velocity dispersion. This bias can be reduced by taking the average of all 120 cross-correlation functions, normalized in intensity, over the whole $5'' \times 8''$ central area: this gives a velocity dispersion $\sigma_p = 11.7 \pm 2.6$ km s^{-1}. This value is independently confirmed by comparing these new observations with numerical simulations. The individual radial velocities of the 14 best-resolved (spatially or spectroscopically) bright stars are also determined; they give $\sigma_p = 14.2 \pm 2.7$ km s^{-1}. These measurements therefore provide no evidence for the velocity dispersion cusp ≥ 25 km s^{-1} ($8.4 \leq \sigma_p \leq 30.0$ km s^{-1}) observed by Peterson et al. (1989).

The above three observed values of the velocity dispersion are complementary to, and consistent with, the work by Gebhardt et al. (1994). In addition, they are all consistent with the predictions of theoretical dynamical models of M15: viz. $\sigma_p(0) =$ 12-17 km s^{-1} from Illingworth & King (1977) using a King-Michie dynamical model, $\sigma_p(0) =$ 13-15 km s^{-1} from Phinney & Sigurdsson (1991) and Phinney (1992, 1993) using the observed decelerations of the two pulsars in the core of M15, and $\sigma_p(0) = 14$ km s^{-1} from Grabhorn et al. (1992) who fitted a Fokker-Planck model to surface brightness and velocity dispersion profiles. Consequently, the presence of any massive black hole or of some non-thermal stellar dynamics in the core of M15 is not required in order to explain the present observations. However, the detection of a moderate velocity cusp — M15 would be the place to look for — would require a better spatial resolution and a sensitivity which are not available yet.

The conclusion of this section is that, contrary to density profiles which provide clear indication of a central density cusp in a few very concentrated globular clusters, no such evidence has been obtained so far from velocity dispersion profiles. So far, core collapse diagnostics is based on density profiles only.

9.4 Physical Interactions

For a long time the study of the dynamics of globular star clusters was one of the "cleanest" theoretical problems in astrophysics, involving nothing more than the gravitational interaction of point masses, albeit in very large numbers. In the 1970s it was held that this was a basic distinction between the evolution of globular clusters and that of galactic nuclei (Saslaw 1973, Bisnovatyi-Kogan 1978). At about the same time it was being realised (Fabian et al. 1975, Finzi 1977) that stellar collisions and close encounters, though they might not affect the overall evolution, could be of importance in understanding the unusual stellar populations in globular clusters, especially the X-ray sources. In more recent

years, however, the realisation has grown (Statler et al. 1987) that inclusion of direct physical interactions between individual pairs of stars may be necessary if theoretical models are to remain reasonably realistic approximations of the dynamical behavior of the entire system. The observational evidence for this is presented in §9.8. Here we summarise recent work on the theory of collisions and other close stellar interactions, though a great deal of other relevant work can be found in literature devoted to galactic nuclei.

To begin with, much of the theory of these processes was semi-analytical, following the technique introduced by Press & Teukolsky (1977); see, for example, Giersz (1986), Lee & Ostriker (1986) and McMillan et al. (1987). The straightforward part of these calculations is the computation of the energy lost in a single encounter, which is what is relevant for the computation of the capture probability. One of the long-standing issues, however, is how to account for the effect of this energy on the internal structure of the participants (Kochanek 1992, Podsiadlowski 1996). If it leads to an expansion of either star, then the effect of subsequent encounters may be collision and coalescence rather than capture. If the tide raised by one star on the other in the first encounter is not dissipated quickly enough (see Kumar & Goodman 1996), then the subsequent evolution of the orbit may be chaotic rather than simply dissipative (Mardling 1995a,b, 1996). A two-body effect that is certainly of importance for close binaries is gravitational radiation (e.g., Buitrago et al. 1994); it might even be detectable in clusters because of its effect on pulsar timings (Sazhin & Saphanova 1993).

In the last few years considerable effort has been expended in the detailed numerical modelling of encounters between stars of different types, in order to measure the cross sections for merging and binary formation, and to measure the amount of mass loss, etc. (Shara & Regev 1986, Soker et al. 1987, Rozyczka et al 1989, Ruffert & Muller 1990, Benz & Hills 1992, Davies et al. 1991, 1992, 1993, Lai et al. 1993, Rasio 1993, 1996a,b, Ruffert 1993, Chen & Leonard 1993, Zwart & Meinen 1993, Lee et al. 1996). In some cases the role of stellar nuclear reactions can be substantial (Benz et al. 1989).

One way in which the importance of encounters may be enhanced is in the context of binary stars. As shown in the next section, interactions involving binaries almost certainly have a major role to play in the overall dynamical evolution, but these interactions themselves will be significantly altered by the finite radii of the stars (Hut & Inagaki 1985). This has been modelled in some detail by Davies et al. (1994). Another circumstance in which the effects of encounters may be greatly enhanced is in cases where the stars are accompanied by circumstellar disks (Murray et al. 1991, Murray & Clarke 1993). At one time, when the evidence for a large population of binaries in clusters was lacking, it was even suggested (Hills 1984) that any primordial binaries might not have survived to the present time because of encounters between the components of a binary at a time when their relative orbit had a high eccentricity.

An important issue in discussions of stellar interactions and mergers is the nature of the objects which will be produced. Such astrophysical implications, and their possible relation with different kinds of more-or-less exotic

stellar populations, have been considered by Bailyn (1988, 1989), Krolik (1983, 1984), Krolik et al. (1984), Ray et al. (1987), and Chen & Leonard (1993). Particular attention has been given to the possible formation by these processes of blue stragglers (Leonard 1989, 1996, Leonard & Fahlman 1991, Leonard & Linnell 1992, Hut 1993b, D'Antona et al. 1995, Lombardi et al. 1995, Eggleton 1996, Rasio 1996a,b), X-ray binaries, recycled pulsars (Ray & Kembhavi 1988), and hot subdwarfs (Bailyn & Iben 1989). The relationships between these different classes of objects are still being argued about. It is possible that the millisecond pulsars in clusters are not descendents of X-ray binaries, for example (Chen et al. 1993). For blue stragglers, the astrophysical issues include whether the encounters can adequately mix the material of the two stars (Rasio 1996a,b), and the luminosity functions of the merger products (Bailyn & Pinsonneault 1995).

How these processes affect a given cluster depends in part, of course, on its density and other parameters. Hills & Day (1976) give a useful tabulation of the expected rates of collisions in a large sample of galactic globular clusters, on the basis of the data available at that time. A more recent study along these lines, emphasising the encounters which could lead to the formation by tidal capture of low mass X-ray binaries, is reported by Verbunt & Hut (1987). When, as in this case, the encounters involve stars of different mass, the results are heavily dependent on dynamical modelling of the clusters (van der Woerd & van den Heuvel 1984, Verbunt & Meylan 1988, Hut et al. 1991). A detailed theoretical study of the rate of formation of cataclysmic variables by tidal capture in two cluster environments is presented by Di Stefano & Rappaport (1994). Sigurdsson & Phinney (1995) have carried out a remarkably thorough investigation of binary-single encounters which takes account of the orbital evolution of the reactants. The results (in terms of the relative and absolute numbers of interesting products, such as blue stragglers, cataclysmic variables and millisecond pulsars) depend on the concentration of the cluster model.

As already stated in the introduction to this section, the other main consequence of these processes is their effect on the cluster itself. Simplified models, based on the energetics of the interactions, were devised by Milgrom & Shapiro (1978), Alexander & Budding (1979), Dokuchaev & Ozernoi (1981a,b), and Giersz (1985a,b).

9.5 Dynamics and formation of binaries

The fact that the central density is predicted to rise to infinity at the end of core collapse (Eq. 9.1) is clear proof of a serious deficiency in the theory. The most likely missing ingredient is binary stars. Even if these are not present initially (primordial binaries) they would form by one or other of the processes which we shall discuss below. Historically, it is these processes of formation which have received most attention, because it was thought for a long time that primordial binaries are essentially absent from globular clusters. This change of

perspective is a fundamental revolution which has not yet been fully absorbed, though the review by Hut et al. (1992a) makes the facts plain.

Binaries are important because of the energy which can be imparted to single stars or other binaries in interactions. This can be understood from several points of view, including thermodynamic arguments (e.g., Horwitz 1981, Padmanabhan 1989b); statistical analyses (e.g., Mansbach 1970, Monaghan 1976a,b, Nash & Monaghan 1978, 1980), since three-body interactions have many chaotic aspects (Boyd 1993); and the study of close triple encounters by the analytical techniques of celestial mechanics (e.g., Marchal 1980) and atomic scattering theory (Grujić & Simonović 1988, Heggie & Sweatman 1991). One of the main tools is the numerical study of the three-body problem. Much interesting data can be found in the papers of the groups at St Petersburg (recent references including Anosova 1986, 1990, 1991, Anosova & Orlov 1988, and Anosova & Kirsanov 1991), at Austin (e.g., Szebehely 1972), and at Turku (e.g., Valtonen 1975, 1976, 1988a; Huang & Valtonen 1987, Valtonen & Mikkola 1991) and in the references mentioned below. Not all of the above data is suitable for statistical analysis of the effects of encounters, however, and sometimes the results are restricted in various ways, e.g., to head-on encounters (zero impact parameter), or circular binaries, though this is an important special case. An ingenious and freely available computational tool for few-body interactions is described by McMillan (1996).

Our knowledge of the energetic effects of three-body interactions in the case of equal masses is reasonably complete (Hills 1975a, Hut 1983, 1993a, Heggie & Hut 1993), and there is extensive information on the case of unequal masses, especially those relevant in applications to globular clusters (Heggie 1975, Hills & Fullerton 1980, Hills 1990, Sigurdsson 1992, Sigurdsson & Phinney 1993, 1995). Less well studied, but also important for investigations of millisecond pulsars, are the effects of encounters on the eccentricity of binary orbits (Hut & Paczynski 1984, Rappaport et al. 1989, D'Amico et al. 1993, Rasio & Heggie 1995, Heggie & Rasio 1996). A fairly comprehensive cross section for exchange involving hard binaries with stars of unequal mass has been provided by Heggie et al. (1996). Of course knowledge of the way in which binaries and stars of different masses behave can only be applied satisfactorily in clusters in which the spatial distribution of the different species is well enough known (Hut et al. 1991).

Energetically these three-body processes are subsidiary to four-body encounters (i.e., binary-binary collisions), where our knowledge of the relevant reaction rates is more patchy, partly because of the greater range of relevant parameters. The most extensive published data has been provided by Mikkola (1983a,b, 1984a,b), and other significant studies have been carried out by Hoffer (1983, 1986) and Hut (1992).

Though those binaries that are dynamically effective are almost certainly outnumbered by single stars in globular clusters, binary-binary encounters are still dominant energetically, for two reasons: (i) the cross section for an energetic interaction with a single star is considerably smaller than that for interaction with another binary; and (ii) it is likely that the mean total mass of the

components of a binary exceeds the mean stellar mass, so that, by mass segregation, they soon become preferentially concentrated in the core (cf. Spitzer & Mathieu 1980), where almost all the energetic interactions take place.

The main effect of these interactions is to halt the collapse of the core. This was demonstrated long ago by Hills (1975b) by means of a very simplified model, though doubt was cast on the effectiveness of binary-binary collisions as a mechanism for halting core collapse by the Fokker-Planck models of Spitzer & Mathieu (1980); their results implied that the halt was only temporary, and that the destruction of binaries in binary-binary encounters (which had not been taken into account by Hills) led quite quickly to continued collapse. However the general picture painted by Hills has subsequently been confirmed, with much additional detail, using a different Fokker-Planck code from that adopted by Spitzer & Mathieu (Gao et al. 1991), though the reasons for the disagreement have never been unearthed. We shall return to the possibility of binary depletion and further core collapse in our discussion of post-collapse evolution (cf. §10.1 below).

These processes can be modelled satisfactorily with N-body simulations (Aarseth 1980, Giannone et al. 1984, Giannone & Molteni 1985, McMillan et al. 1990, 1991, McMillan 1993, McMillan & Hut 1994, Heggie & Aarseth 1992, Aarseth & Heggie 1993, Kroupa 1996), all of which have confirmed that core collapse can indeed be brought to an end, and have demonstrated how the point at which collapse is halted is affected by the parameters of the binary distribution (mainly the range of internal energies, and their numbers). One of the uncertainties here, however, is the distribution of masses of binary components.

We have pointed out that binary-binary interactions are effective in halting the collapse of the core. The other main effect of interactions involving binaries is the effect on the participating stars and binaries, and even on putative planetary companions of pulsars (Sigurdsson 1992)! As already mentioned, collisions are effective in destroying binaries (the outcome of most binary-binary interactions being the destruction of one participant), in hardening those that remain, and in ejecting them to the outer parts of the cluster. The effect of three-body interactions on the distribution of (internal) binding energies was studied, in the context of a homogeneous stellar system, by Retterer (1980b) and, using better scattering cross sections, by Goodman & Hut (1993). In fact, however, the hardening of the binaries also influences their spatial distribution, as was clearly demonstrated in a simplified model by Hut et al. (1992a). For example, such interactions may be needed in order to understand the spatial distribution of pulsars (Phinney & Sigurdsson 1991) and blue stragglers (Sigurdsson et al. 1994). In slightly more extreme cases triple interactions may lead to high-velocity escapers (cf. §7.3), and it is just possible that two rapidly moving stars in M3 (Gunn & Griffin 1979) as well as in 47 Tucanae (Meylan et al. 1991a) originated in this way. A further effect of interactions involving binaries is exchange reactions, which is thought to be the main channel for the formation of X-ray sources. Examples of specific systems whose dynamical evolution has been studied with regard to triple interactions are provided by Rasio et al. (1995) and Sigurdsson (1993).

Fig. 9.7. Hydrodynamic simulation of a binary-binary encounter (from Goodman & Hernquist 1991 Fig. 2d). The two binaries are shown emerging from an encounter, which so perturbs the upper pair that they coalesce. The merger remnant is described as a rapidly rotating spheroidal star surrounded by a thick disk. No collision occurs in a simulation without hydrodynamics. Initial conditions: binary orbits are circular with random orientations, and semi-major axis a; relative orbit of the binaries is parabolic; stars are equal-mass polytropes of index $n = 3/2$ and radius $a/6$.

Though one of the participants is likely to be destroyed in any close encounter between two binaries, this is a significant route for the formation of relatively long-lived multiple systems (Kiseleva et al. 1996). The stability of such systems is a long-standing issue in dynamical astronomy (see, for this and many other aspects of the three-body problem, the book by Marchal 1990),

and clearly can have profound implications for the internal evolution of the member stars. This is one way in which the dynamical evolution of globular clusters becomes bound up with the way in which the stars themselves evolve, and it has stimulated renewed interest in the stability and stellar evolution of triple systems (Kiseleva et al. 1994, Eggleton & Kiseleva 1995).

The main unexplored complication in all this is the finite radii of the stars taking part in these interactions. It has been pointed out (Aarseth, pers. comm.) that the N-body models predict their own downfall by confirming theoretical expectations that stellar collisions should be frequent. As mentioned in the previous section, the effects of collisions on the details of individual encounters are dramatic (McMillan 1986a, Cleary & Monaghan 1990, Goodman & Hernquist 1991; cf. Fig. 9.7). As yet, however, little has been done to follow through the consequences for core evolution, though this is within the scope of suitable codes, using either "sticky" particles or smooth particle hydrodynamics (SPH), except in the context of open clusters (Aarseth 1992, 1996a,b). Though the astrophysical complications are great, so are the astrophysical pay-offs. Possible consequences of interactions involving binaries with stars of finite size are enhanced mass transfer (Shull 1979), the formation of blue stragglers (e.g., Bailyn 1992, Bacon et al. 1996), and the observed depletion of red giants in cluster of high concentration (e.g., D'Antona et al. 1995). Davies (1995) has shown how a remarkable variety of astrophysically realistic interactions may be modelled quite economically, and this has been applied to 47 Tucanae and ω Centauri by Davies & Benz (1995).

Now we briefly turn to some older studies involving the formation of binaries in a system without primordial binaries. Two mechanisms were considered (see, for example, Giersz 1984 for unified treatments). The less dominant in most conditions (Dokuchaev & Ozernoi 1978, Inagaki 1984) is the formation of binaries in three-body interactions, which was essentially discovered in N-body simulations of van Albada (1967) and Aarseth (1968, 1972, 1977, 1985b). Quite simple estimates (e.g., Heggie 1984) successfully predict that such binaries can arrest core collapse, though usually some other process intervenes first: in almost all real systems without primordial binaries it would be dominated by the two-body (tidal) formation mechanism (Fabian et al. 1975, Press & Teukolsky 1977, Lee & Ostriker 1986; this review, §9.4), except in the case of a cluster core dominated by degenerate stars (Lee 1987a). Thousands of binaries may form from this mechanism, and part of its continuing importance is that a large fraction would form before core collapse is well advanced. Therefore this is still an important mechanism even if core collapse is arrested at relatively low densities by primordial binaries. What complicates the problem is that tidal binaries are extremely tight, and any interaction is likely to lead to collision. Furthermore the number of pairs is likely to be matched by the number of stellar collisions. Therefore the production of coalesced stars may be the main outcome of this mechanism, but, despite its potential importance for giving rise to exotic types of star (cf. §9.7 below) its interaction with dynamics has been little studied in recent years, except in the context of post-collapse expansion (§10.1).

9.6 Observational evidence of binaries in globular clusters (compared to the field)

Although there is now clear and plentiful observational evidence of binaries in globular clusters, this has not always been the case. In spite of intensive searches for some decades preceding the mid eighties, i.e., before the use of CCDs in astronomy, there was no known photometric or spectroscopic binary in globular clusters. Gunn & Griffin (1979), who did not discover any spectroscopic binaries among the 111 stars they observed in M3, with radial velocities to an accuracy of $\sim 1\ \mathrm{km\,s^{-1}}$, concluded that binarism involving stars with separations in the range 0.3-10 AU is either very rare or absent in globular clusters, in stark contrast to the situation in the solar neighborhood and in open clusters. Fifteen years later, it is now clear that the usual sort of photometric and spectroscopic binaries, with periods from one day to one year, do exist in globular clusters, in addition to more exotic objects like binary milli-second pulsars.

Binary star formation scenarios may be categorised in a variety of ways, but the most fundamental distinction is between those in which stars form singly within a cluster and subsequently pair up (as described in §§9.4 and 9.5 above), and those in which stars form as binaries as a result of a splitting into two during the star formation process (cf. §5.4 above). From a purely observational point of view, it is impossible to disentangle the class of model (capture and fragmentation) by which a given observed binary star has been formed, although soft (long-period) binaries may be preferentially primordial and hard (short-period) binaries may be of more recent formation.

This section describes the observational evidence for binaries provided by photometric and radial-velocity surveys, without mentioning the possible origin scenario. The ultimate aims of such studies are (i) to establish the frequency of binaries and (ii) to determine their radial distribution within the cluster, two quantities which are intimately linked to the internal dynamics. See also the reviews by Hut et al. (1992a) and Phinney (1996).

Observational evidence of binaries from photometry. For a variety of practical reasons, most searches for photometric binaries in globular clusters have been restricted to the study of short-period ($\lesssim 5$ days) eclipsing binaries and cataclysmic variables only. For several decades, all eclipsing binaries observed in globular clusters turned out not to be members, once the appropriate radial velocity information was acquired.

The first clear case of a genuine member was obtained by Niss et al. (1978), who identified in ω Centauri one certain eclipsing binary (NJL 5, with P = 1.38 day), whose membership was subsequently confirmed, on the basis of radial velocity measurements, by Jensen & Jørgensen (1985) and Margon & Cannon (1989). Since then, the list of such stars has increased steadily, with potential of giving insight into the frequency of binary stars. For example, Mateo et al. (1990) discovered, from a long series of CCD exposures, three

eclipsing systems among the population of blue stragglers in NGC 5466, one of them being of Algol type, the two others of W UMa type. Yan & Mateo (1994) found, in the centre of M71, five binaries, one of them being of Algol type, the four others of W UMa type. See Yan & Reid (1996) for a search in M5 and Yan & Cohen (1996) for NGC 5053. Since binaries are expected to be more numerous in the centre, due to mass segregation, photometric searches from the ground are limited to the loose globular clusters like ω Centauri or to the outer parts of concentrated clusters like 47 Tucanae. E.g., Rubenstein & Bailyn (1996) have discovered a W UMa binary in the globular cluster NGC 6397, at about $2'$ from the centre. A very interesting by-product of the Optical Gravitational Lensing Experiment (OGLE) is the discovery, apart from SX Phe pulsating stars, of eclipsing binaries in ω Centauri and 47 Tucanae (Kaluzny et al. 1996a,b,c).

Fig. 9.8. Cumulative radial distributions, of the binary stars, of the blue stragglers (BSs), and of all main sequence, subgiant, and giant stars with the same magnitude range ($15.9 < U < 20.4$) as the binary stars detected in 47 Tucanae (from Edmonds et al. 1996 Fig. 15).

With HST, similar searches are possible even in the cores of the most concentrated globulars. Gilliland et al. (1995) and Edmonds et al. (1996) have monitored 20,000 stars in the core of 47 Tucanae using differential time series

U photometry with the WFPC1. Using aperture photometry, PSF fitting, and power spectrum techniques, they discovered 2 W UMa binaries in addition to 6 semi-detached or detached binaries with periods between 0.41 and 1.5 days.

Fig. 9.8 displays, in the case of 47 Tucanae, the cumulative radial distributions of the binaries discovered by Edmonds et al. (1996), the cumulative radial distributions of the blue stragglers discovered by Guhathakurta (1996a), and of all the other stars in the same range of magnitude. It appears that the binaries and the blue stragglers have similar radial distributions and that both are more centrally concentrated than the normal stars (see also Guhathakurta et al. 1992). This result implies that, in 47 Tucanae, the binaries, compared to other stars, are more centrally concentrated than the binaries in M71 (Yan & Mateo 1995) and in NGC 4372 (Kaluzny & Krzeminski 1993). This may be related to the different concentrations and central relaxation times of these clusters, although definitive statements will be made possible only when a complete census of binary stars, down to a given limiting magnitude, will be available.

It is worth mentioning that the present samples obtained in different clusters are very dissimilar, covering different period ranges, at different distances from the centres of the clusters.

Observational evidence of binaries from radial velocities. So far, virtually all radial velocity surveys of stars in globular clusters have been related to the luminous giants. Their large radii, typically 0.1-0.4 AU, impose a bias on the periods detectable. Binary systems with periods shorter than about 40 days, i.e., with separations less than about 0.25 AU, will not reach the luminosities required to be included in the magnitude-limited samples because they will have suffered mass transfer which either truncates the evolution of the giant or leads to coalescence through a common-envelope stage (Pryor et al. 1988). This bias towards long periods means that improved velocity precisions are needed to discover shorter period binaries.

A typical giant primary of 0.8 $M_\odot$ with a companion of 0.4 $M_\odot$, separated by 0.25 AU, gives a binary with a period of 42 days; if the orbit is circular, the giant star has an orbital velocity of 22 $\mathrm{km\,s^{-1}}$. Increasing the period by a factor of 10 increases the separation to 1.2 AU and decreases the velocity to 10 $\mathrm{km\,s^{-1}}$. Consequently, detecting such binaries requires studies lasting years and velocity measurements accurate to about 1 $\mathrm{km\,s^{-1}}$. Such velocity precisions have been achieved, for about two decades now, by the two CORAVEL spectrometers, the Dominion Astrophysical Observatory radial-velocity scanner, and by the intensified Reticon system of the Smithsonian Astrophysical Observatory.

The conclusion by Gunn & Griffin (1979) that binarism is either very rare or absent in globular clusters, was criticized by Harris & McClure (1983). They pointed out that, giant stars in globular clusters having lower masses and larger radii than field Population I giants, the results of Gunn & Griffin (1979) were compatible with the Abt & Levy (1976) field binary frequency. This prompted D. Latham and T. Pryor to undertake extensive new observations of

the M3 giants which resulted in the discovery of the first spectroscopic binary in a globular cluster, vZ 164 (Latham et al. 1985). The list of such stars has increased steadily since then.

A major search for binaries in the globular cluster NGC 3201 has been published by Côté et al. (1994) who obtained multiple velocity measurements for 276 stars, with a mean time span between observations of 1.7 year, and with coverage up to about 6 years for the best studied stars. They find 21 stars with some signature of binarity, although the radial velocity measurements of some of these binary candidates, which are among the brightest cluster members, may suffer from the so-called "jitter" due to stellar atmospheric motions, first described by Gunn & Griffin (1979) in M3; see also Mayor et al. (1984) in 47 Tucanae.

Another major study, from the point of view of both sample size and time baseline, concerns the giant galactic globular cluster ω Centauri (Mayor et al. 1996). It is worth mentioning that ω Centauri, being an old globular cluster, is perfectly well suited to provide information on primordial binaries since the characteristic time for dynamical evolution of spectroscopic binaries (with periods $P < 30$ yr) is much longer than the cluster age. The rather low central stellar density of this loose globular cluster and its related large half-mass relaxation time ($26 \leq t_{rh} \leq 46 \times 10^9$ yr: Meylan et al. 1995) ensure that dynamical influences on the primordial binary population through close stellar encounters have not been great. Actually, the present period range for primordial binaries among red giants is limited on the short-period side by the onset of Roche-lobe overflow, and on the long-period side by dynamical disruption. Consequently, most of the primordial binaries among the giant of ω Centauri are expected to have periods from 200 to 4,000 days.

Between 1981-1993, the radial velocities of 310 giant stars which are members of ω Centauri were monitored by Mayor et al. (1996), with a mean error of a radial velocity measurement of about 0.7 km s^{-1}. All stars have 3 or more measurements. The "jitter" observed in the radial velocities of bright giant stars in globular clusters can be easily disentangled from the variations due to spectroscopic binaries since their effects on the observed cumulative distribution of the standard deviations are quite distinct. Two stars are definite binaries.

Côté & Fischer (1996) have undertaken a search for spectroscopic binaries on the main sequence of the nearby globular cluster M4. A pair of radial velocities (median precision $\simeq 2$ km/s) separated by 11 months have been obtained for 33 turn-off dwarfs in the magnitude range $16.9 \leq \mathrm{V} \leq 17.4$.

Côté et al. (1996) report on search for long-period binaries in M22. They use observations accumulated between 1972 and 1994. This 22-year baseline is the longest available for any sample of globular cluster stars. Using 333 repeat velocities for 109 cluster members, they search for spectroscopic binaries with periods in the range 0.2 to 40 years and with mass ratios between 0.1 and 1.0.

On the frequency of binaries. Contrary to what was thought in the early 80's, we now know that binaries do exist in globular clusters. The observational evidence comes through two different routes: (i) from photometric light curves, which are efficient in discovering short-period (detached, semi-detached and contact) binaries, with data acquired within a few nights, and (ii) from radial velocity curves, which are efficient in discovering long-period (primordial) binaries, with data acquired over more than a decade.

The ultimate aim of all searches for binary stars in globular clusters is the knowledge of the frequency of binaries (is it higher, equal, or smaller than in the solar neighborhood?), since, through their formation and destruction, these stars play a fundamental role in the dynamical evolution of these stellar systems, especially during core collapse phases (cf. §§9.1 and 9.2 above). Unfortunately, this is not an easy task given the very large diversity of non exhaustive surveys, sampling different period ranges, and the fact that the binary frequency may vary from one cluster to the other, and from one period range to the other. The only recourse for estimating the binary frequency consists of comparing data with large numbers of simulations.

Considering the data from (short-period) eclipsing binary surveys and adopting from Duquennoy & Mayor (1991) a binary fraction in the solar neighborhood of 65%, Hut et al. (1992a) conclude that the overall binary fraction in globular clusters is between 20% and 35%, i.e., significantly lower than the frequency in the solar neighborhood. In the case of radial velocity surveys, for binaries with $0.2 \leq P \leq 20$ yr and $q \leq 0.22$, Hut et al. (1992a), quoting Pryor et al. (1989b), argue for a fraction between 5% and 12%, i.e., at most a small deficiency of binary stars in globular clusters when compared to the solar neighborhood.

From their photometric survey, Yan & Mateo (1994) determine a lower limit of 1.3% on the fraction of primordial binaries in M71 with initial orbital periods in the range 2.5-5 days. From their simulations, they conclude that this implies an overall primordial binary frequency $f = 22\,^{+26}_{-12}\%$ assuming $\mathrm{d}f/\mathrm{dlog}\,P = \mathrm{Cst}$ (the "flat" distribution) or $f = 57\,^{+15}_{-8}\%$ assuming $\mathrm{d}f/\mathrm{dlog}\,P = 0.032 \log P + \mathrm{Cst}$ (the "sloped" distribution) as observed for G-dwarf binaries in the solar neighborhood (Duquennoy & Mayor 1991). In the case of M5, Yan & Reid (1996) estimate an overall primordial binary frequency $f = 28\,^{+11}_{-5.8}\%$ assuming $\mathrm{d}f/\mathrm{dlog}\,P = \mathrm{Cst}$ (the "flat" distribution) for the period range 2.5 days to 550 years. Yan & Cohen (1996) obtain a binary frequency in NGC 5053 equal to 21-29% with $3\mathrm{d} < P < 10\mathrm{yr}$, $0.125 < \mathrm{q} < 1.75$. This somewhat higher estimate is perhaps related to the fact that NGC 5053 is relatively dynamically young when compared to other clusters. See also Yan (1996).

Edmonds et al. (1996), from considerations related to their observed numbers of W UMa and Algol systems and to orbital angular momentum loss theory, conclude that the population of binaries detected photometrically in the core 47 Tucanae appears fundamentally different from populations discovered in other globular and open clusters. They argue that at least some of these binary systems have been formed in the cluster core through stellar encounters.

In relation to their radial velocity survey, Côté et al. (1994) constrain the

frequency of binaries in NGC 3201, from exhaustive Monte Carlo simulations. Assuming a thermal distribution of eccentricities, for periods $0.1 \leq P \leq$ 5-10 yr and mass ratio $0.1 \leq q \leq 1$ they obtain a fraction of binaries $\lesssim 15\%$ - 18%. For binaries with circular orbits, these limits fall to 6% - 10%. Consequently, the binary fraction in NGC 3201 appears equal to, or slightly higher than, that of the field which is equal to 4% - 8% in a comparable range of period and mass ratio.

Assuming that ω Centauri has a period distribution similar to the one observed for the nearby G dwarfs (Duquennoy & Mayor 1991), Mayor et al. (1996) estimate the global binary frequency in ω Centauri to be as low as 3-4%, significantly smaller than the 13% of binaries with $P <$ 3,000 days found among the nearby G dwarfs by Duquennoy & Mayor (1991).

From the turnoff main-sequence stars observed in M4, Côté & Fischer (1996) find, using Monte-Carlo simulations, a binary fraction of 15% for systems with periods in the range from 2 days to 3 years and mass ratios between 0.2 and 1.0. From the giant stars observed in M22, Côté et al. (1996) find, using Monte-Carlo simulations, a binary fractions between 1-3%, results consistent with Mayor et al. (1996).

The studies of Mayor et al. (1996) and Côté et al. (1996) point towards a fraction of primordial binaries in ω Centauri which is significantly smaller than the fraction of primordial binaries in the solar vicinity and in open clusters. This is at variance with, e.g., the results obtained by Pryor et al. (1989b), Hut et al. (1992a), and Côté et al. (1994) for other globular clusters. This may be either the results of intrinsic differences between the studied clusters or the consequences of differences in the simulations and their interpretation. These simulations are in all cases sophisticated, complicated by numerous assumptions and astrophysical inputs, which make their analysis subtle and their comparison with the observations not entirely straightforward. Côté et al. (1996) speculate that both the relative abundances of short- and long-period binaries in globular clusters and the large differences in measured binary fractions for clusters with high binary ionization (i.e., disruption) rates (M22, Omega Cen) compared to those for clusters with low ionization rates (M71, M4, NGC 3201) point to a frequency-period distribution in which soft binaries have been disrupted by stellar encounters.

9.7 Influence of dynamical evolution on stellar populations

It is now commonly accepted that globular cluster stellar populations exhibit numerous observable scars which betray the influence of stellar dynamics on stellar populations. The straightforward observation concerns colors of stars and, consequently their positions in the color-magnitude diagram. The idea of linking macroscopic (dynamical evolution of a cluster as a whole) and microscopic (stellar evolution of a single star) phenomena is rather recent. And it acts both ways: the general dynamical evolution of the cluster can influence

the fate of a single star, but the presence, e.g., of a few binaries in the core, can also influence the dynamical evolution of the cluster as a whole.

Blue horizontal-branch stars and blue subdwarfs. Suspicions that stellar dynamics may influence the stellar evolution in globular clusters are more than a decade old (Renzini 1983). E.g., Buonanno et al. (1985a), in their study of the giant, asymptotic, and horizontal branches in color-magnitude diagrams of globular clusters, mentioned the very different dynamical status of M15 and NGC 5466 as the possible reason for the presence or not of faint blue horizontal branch stars. From an age and metallicity point of view, these two clusters are as similar as they could be, but they strongly differ only in relation to their structural parameters and central densities, as presented in Table 9.1.

Table 9.1: Comparison between M15 and NGC 5466

parameter	NGC 7078 M15	NGC 5466
age τ	16 ± 3 Gyr	16 ± 3 Gyr
helium abundance Y	0.23 ± 0.02	0.23 ± 0.02
metal abundance [Fe/H]	-2.10 ± 0.20	-2.05 ± 0.20
[O/H]	-1.3 ± 0.3	-1.3 ± 0.3
concentration $c = \log\,(r_t/r_c)$	2.8	1.5
central density $\rho_\circ$	$1.6 \times 10^6\ M_\odot\ \mathrm{pc}^{-3}$	$6.3 \times 10^0\ M_\odot\ \mathrm{pc}^{-3}$

M15 is a very concentrated globular cluster, considered as a prototype of collapsed clusters (cf. §§9.1, 9.2, and 9.3 above), while NGC 5466 is a rather loose cluster. The former should suffer large numbers of stellar collisions in its core, contrary to the latter. A high rate of encounters and collisions should produce, through coalescence and merging, numerous stars heavier and bluer than turn-off stars. As conspicuously visible in Fig. 9.9, the color-magnitude diagrams of M15 and NGC 5466 differ only by the presence of a large number of blue stars at the left end of the horizontal branch. They are called blue subdwarfs and, in globular clusters, also referred to as extreme or faint blue horizontal branch stars, since they form a vertical continuation to the horizontal branch.

Given the fact that, in a color-magnitude diagram, the faintest blue subdwarfs constitute a downwards extension of the horizontal branch, they can be mixed and confused with the brightest blue stragglers, which form an upwards extension of the main sequence, above the turn-off. Despite their proximity, blue subdwarfs and blue stragglers are thought to be very different, the former being closely related to horizontal branch stars.

From the examination of the horizontal branch structure of 53 clusters, Fusi Pecci et al. (1993a) find that the length of the blue tail of the horizontal branch correlates with cluster density. Recent HST observations with

WFPC2 of ten galatic globular clusters have resulted in the first discovery of hot horizontal-branch stars in two metal-rich clusters, NGC 6388 and NGC 6441 (Rich /etal/ 1996) and in the discovery of an intriguing multimodal horizontal branch in NGC 2808 (Sosin /etal/ 1996).

Fig. 9.9. Upper panel: Color-magnitude diagram for all the stars in M15 (NGC 7078) brighter than $B = 18.6$ in the annulus with radii $1.9' < r < 5.0'$. Variables and fields stars have been omitted. Lower panel: Color-magnitude diagram for all the stars in NGC 5466 brighter than $B = 19.0$ in the annulus with radii $0' < r < 5.5'$. Variables and fields stars have been omitted. All data are from Buonanno et al. (1985a).

In ω Centauri, there is evidence of segregation, towards the cluster centre, of blue subdwarfs with respect to other stars (Bailyn et al. 1992), but Drukier et al. (1989) find in M71 that blue subdwarfs are less concentrated than other

giants. A decrease in the frequency of blue subdwarfs towards the core of M15 is also observed by Buonanno et al. (1985a) and De Marchi & Paresce (1994b). Mass segregation only cannot be used to account for such a diversity of behavior. Ways of producing blue subdwarfs from binaries have been investigated by Iben & Tutukov (1986) and Bailyn & Iben (1989).

Color gradients. Early reports of color gradients in globular clusters have illustrated the difficulty in providing conclusive observations of such phenomena. E.g., the results by Chun & Freeman (1979), who found cluster integrated light becoming redder towards the centre, were subsequently shown to be the consequences of clumps of red giants stars more or less centered on the aperture, whose position can also slightly vary between two different bands (Buonanno et al. 1981). Since then, CCDs have allowed the clear observation of color gradients in some of the galactic globular clusters, although these gradients appear in the sense that cores are bluer than the outer regions. See Djorgovski & Piotto (1993) for the most important review on this subject.

Two different and largely complementary methods have been used:

• Piotto et al. (1988) employed a generalization of a standard surface photometry technique (described by Djorgovski 1988) to simultaneously measure multiple color frames of the collapsed cluster M30. They found that this cluster becomes bluer towards the centre. This gradient is present in different sets of data, is significant at a 10-σ level, and amounts typically to $\Delta(B-V)/\Delta\log r \sim 0.15$ mag and $\Delta(B-R)/\Delta\log r \sim 0.25$. This direct surface photometry technique is mainly sensitive to effects present among the bright stars, which contribute most of the total light. Djorgovski et al. (1989) found a color gradient in NGC 6624, another collapsed cluster, but not in NGC 6093, a King-model cluster. Djorgovski et al. (1991) extended this study to 12 clusters, confirming the trend that color gradients are present in collapsed clusters but not in King-model clusters. This points towards a link between color gradients and the general dynamical evolution of a cluster as a whole.

• Bailyn et al. (1988, 1989), using a complementary technique based on pixel histograms, discovered a gradient in the collapsed cluster M15. They claimed that this gradient is caused by some intrinsically faint stellar population, since their technique is mainly sensitive to the effects present among the faint and numerous stars which cover most of the detector area.

This apparent contradiction between the results from the two different techniques is believed to be due to some genuine difference in the nature of gradients in different clusters. Cederbloom et al. (1992) and Stetson (1991, 1994), with excellent CFHT data, confirm the color gradient in M15. The latter concludes that it is due to three different effects: (i) a deficiency of the brightest red giants in the cluster centre, (ii) the giant branch shifts towards the blue as the centre of the cluster is approached, (iii) the very centre of the cluster contains a large population of blue stragglers, many of them with a significant ultraviolet excess (see also Aurière et al. 1990, Djorgovski et al. 1991, Djorgovski & Piotto 1992). The presence of very blue stars in the core of

M15 and NGC 6397 may also have some causal links to stellar dynamics in the core of this cluster (De Marchi & Paresce 1994a,b, 1995b, 1996); this idea was already mentioned by Dupree et al. (1979) and Djorgovski & Piotto (1992).

Although exhibiting a large variety in their characteristics, color gradients are observed in all collapsed/high-concentration globular clusters in which they have been looked for, but not in King-model clusters. When present, a gradient is always in the sense of the color becoming bluer towards the centre, and it starts in radius at about $20''$ to $100''$ from the centre. In some clusters, e.g., M30, color gradients seem to be only the consequences of differences in the distributions of bright stars (Piotto et al. 1988, Djorgovski et al. 1989, and Burgarella & Buat 1996). In other clusters, e.g., M15, the gradients seem to be due mainly to the fainter unresolved stars (Bailyn et al. 1988, 1989 and Cederbloom et al. 1992). And in a third group of clusters, e.g., NGC 6397, there are color gradients in the light from both bright and faint stars (Lauzeral et al. 1993 and Djorgovski & Piotto 1993). In some clusters, e.g., M30, M15, and NGC 6397, there is a clear depletion of bright red giants near the cluster centre rather than an increase in the numbers of horizontal branch stars. See also Bailyn (1994) in the case of 47 Tucanae. The morphology of the horizontal branch is correlated with the cluster central density and/or concentration (Renzini 1983), in the sense that denser and more concentrated clusters, like M15 (see Fig. 9.9 above) tend to have a more extended horizontal branch with a faint blue tail (Fusi Pecci et al. 1993a). A possible explanation of this phenomenon may be that the red horizontal branch stars near the centre could be a progeny of blue stragglers (Fusi Pecci et al. 1992).

9.8 Observational evidence of possible products of stellar encounters (blue stragglers, high-velocity stars, X-ray sources, and pulsars.)

Blue stragglers. Blue stragglers were first observed by Sandage (1953) in the globular cluster M3, as a bunch of stars forming an upwards extension of the main sequence, above the turn-off, in the usual color-magnitude diagram. During the following 3 decades, blue stragglers were discovered, although at a rather slow pace, among the halo field stars, in young and old open clusters, as well as in globular clusters. A resurgence of interest in the search for blue stragglers in globular clusters was initiated by Nemec & Harris (1987), who, using CCDs and software for photometry in crowded fields (cf. §6.1), discovered blue stragglers in NGC 5466. Among others, two important studies are by Aurière et al. (1990), who observed blue stragglers in the dense collapsed core of NGC 6397, and by Mateo et al. (1990), who discovered three eclipsing binaries, with periods between 0.298 and 0.511 day, among the nine variable blue stragglers in NGC 5466. But it is essentially since the launch of the HST, whose spatial resolution allows the easy detection of blue stragglers even in the crowded cores of globular clusters, that there has been a flurry of discovery papers, starting with Paresce et al. (1991), who observed blue stragglers in the

core of 47 Tucanae.

Every appropriate search in any globular cluster has unveiled blue stragglers: they are ubiquitous. Any new search unveils more blue stragglers (e.g., Burgarella et al. 1995 in M3). Catalogs of blue stragglers in globular clusters have been published by Fusi Pecci et al. (1992, 1993b), and lists of globular clusters with blue stragglers are given in Sarajedini (1993) and Ferraro et al. (1995a). There are now more than 800 blue stragglers known in more than 30 globular clusters. Unfortunately, statistics about blue stragglers are difficult to extract from the data which vary strongly from cluster to cluster: photometric filters (from UV to IR), location in the cluster with respect to its centre, area surveyed, limiting magnitudes, are as different as they can be, given the intrinsic differences between ground-based and HST data.

It is now convincingly demonstrated, from radial cumulative distributions, that blue stragglers are more centrally concentrated than the other stars of same magnitudes (see Fig. 9.8 above). This effect was first observed in NGC 5466 by Nemec & Harris (1987), and subsequently in numerous other clusters (e.g., Lauzeral et al. 1993; see Bailyn 1995 for a review). The central concentration of the blue stragglers is considered as a consequence of mass segregation, since the central relaxation time is always significantly smaller than the lifetime of stars of masses in the range 1.0-1.5 $M_\odot$. Consequently, when present in a cluster, such stars should be more centrally concentrated than the most luminous stars – giants and subgiants – whose masses are about 0.8 $M_\odot$. Nemec & Harris (1987) derived, by comparison with multi-mass King models, a mean mass of 1.3 ± 0.3 $M_\odot$ for the blue stragglers in NGC 5466, in agreement with what would be expected from their position in the color-magnitude diagram. There is, however, the noticeable exception of M3, where there is an excess of blue stragglers in the inner and outer regions, and a lack of blue stragglers at intermediate radii (Ferraro et al. 1993, Bolte et al. 1993, Guhathakurta et al. 1994). This could be due to the existence of two different populations of blue stragglers within the same cluster or to segregation effects in the production and/or survival of blue stragglers (see Davies et al. 1994, Sigurdsson et al. 1994).

It is clear, from the paper by Mateo et al. (1990), that blue stragglers represent a very heterogeneous family: e.g., in NGC 5466, a fraction of the blue stragglers are variable in luminosity, some being pulsating stars and others being eclipsing binaries of W UMa and Algol types. Consequently, more than one scenario may be at work in order to provide this diversity (Livio 1993). The numerous different models can be divided into two groups: (i) models involving single stars and (ii) those involving binaries (Livio 1993, Stryker 1993, Ouellette & Pritchet 1996). Hereafter we mention only the most plausible scenarios:

(i-a) Multiple bursts of star formation: this scenario may be at work in young populations ($\tau \lesssim 10^8$ yr) for which isochrones in a color-magnitude diagram reveal gaps which could indicate that stars with masses $M \geq 5$ $M_\odot$ were formed in a more recent burst of star formation (Eggen & Iben 1988). In older open and globular clusters, however, such a delayed formation scenario would require implausibly large quantities of gas long after the first generation of stars.

(i-b) Internal mixing: Wheeler (1979) suggested internal mixing as a mechanism to extend the main sequence lifetime of stars. The reason for mixing is not clear, although rotation and magnetic fields have been mentioned (Maeder 1987). From preliminary results from stellar evolutionary codes including rotation, the lifetime of a main sequence star could be significantly increased (Maeder & Meynet pers. comm.). Tidal interaction may be another way to induce mixing, although it is related to scenarios involving binaries instead of single stars. An interesting method to test for the mixing hypothesis through lithium abundance has been suggested by Pritchet & Glaspey (1991).

(ii-a) Mass transfer in binaries without coalescence: the blue stragglers would increase their masses via mass transfer in close binaries (McCrea 1964, van den Heuvel 1994). A clear prediction of this scenario is that all blue stragglers should be in binaries, in clear contradiction with observations (e.g., Milone & Latham 1992).

(ii-b) Coalescence in binaries: the suggestion that some blue stragglers are coalesced binaries is due to Zinn & Searle (1976). This would be the end result of contact binaries (van den Heuvel 1994) or evolution through a common envelope phase (Meyer & Meyer-Hofmeister 1980). In the case of NGC 5466 (Mateo et al. 1990), comparison between the numbers of close binaries which are blue stragglers and the expected numbers, based on the time scale required for contact binaries to merge and the lifetime of blue stragglers, leads to the conclusion that not all blue stragglers in NGC 5466 formed by coalescence. See Livio (1993) for a summary of a few observational facts leading to this conclusion.

(ii-c) Mergers during dynamical interactions (encounters and collisions): the improvements during the last two decades in the understanding of the dynamical evolution of globular clusters have emphasized the essential role of tidal captures and collisions in the core of these stellar systems (cf. §§9.4 and 9.5 above). On the basis of these mechanisms, Krolik (1983) predicted that globular clusters should contain substantial numbers of close binaries, contact binaries, and blue stragglers, with their origins in encounters or collisions. Mergers induced by collisions have been studies by Benz & Hills (1987), Leonard (1989), Leonard & Fahlman (1991), Leonard & Linnell (1992), Lombardi et al. (1995), and Leonard & Livio (1995), among others (cf. Fig. 9.7). There is still ambiguity about whether blue stragglers are single or double stars simply because of the possibility that some of them have merged, although it is clear that the abundance of binaries among blue stragglers is unusually high (Mateo 1996). Ambiguity is also present in the prediction of the rotational speed of a merger product: Leonard & Livio (1995) find that rapid rotation is not a signature of a collisionally merged blue straggler. Equally ambiguous is the degree of mixing, since blue stragglers formed by direct stellar collisions are not necessarily fully mixed and not expected to have anomalously high helium abundances in their envelopes, or to have their cores replenished with fresh hydrogen fuel (Lombardi et al. 1995, Procter et al. 1996). When applied to the Yale Rotating Evolution Code in order to explain the six central bright blue stragglers in the core of NGC 6397, these models predict that the collision products must be ei-

ther more than twice the turn-off mass or mixed by some process subsequent to the initial collision and merger (Sills et al. 1996; see also Bailyn & Pinsonneault 1995).

There is a body of evidence which favor the idea that binaries, via various mechanisms (viz., interaction, capture, coalescence, merging), are related to the origin of some blue stragglers in galactic globular clusters, although, from both observational and theoretical points of view, the picture is still vague. Fusi Pecci et al. (1992) and Ferraro et al. (1995a) attempted to extract synthetic information from the currently available surveys of blue stragglers in all observed clusters. E.g., in loose clusters the number of blue stragglers detected so far seems to increase almost linearly with the amount of sampled light, while the trend changes abruptly for clusters having intermediate and high concentrations. The fact that highly concentrated globular clusters have far fewer blue stragglers per unit of luminosity than loose globulars may be simply the consequence of the greater difficulty in detecting them in dense cores (Ferraro et al. 1995a, Kaluzny et al. 1996c). If it is a genuine effect, it may be that blue stragglers in loose clusters originate from primordial binaries while those in high density clusters are produced by stellar interactions (Bailyn 1992; Bailyn & Pinsonneault 1995; Ferraro et al. 1995a).

High-velocity stars. Two independent studies, based on radial velocities of individual stars in globular clusters, have discovered stars with unexpectedly high velocities.

Gunn & Griffin (1979) were puzzled by two stars that they called "interlopers". These are two high-velocity stars (viz. von Zeipel 764 and 911) located in the core of M3 $\equiv$ NGC 5272, both about $20''$ from the centre. They have radial velocities relative to the cluster of +17.0 km s^{-1} and –22.9 km s^{-1}, corresponding to 3.5 and 4.5 times the velocity dispersion in the core, which is $\sigma_p(\mathrm{core}) = 4.9$ km s^{-1}. These radial velocities are still close enough to the mean radial velocity of the cluster to carry a strong implication of membership, since the cluster velocity is $V_r \simeq -147$ km s^{-1}, high enough to make contamination by field stars very unlikely. In a similar way, Meylan et al. (1991a) discovered two high-velocity stars in the core of the globular cluster 47 Tucanae. Located respectively at about $3''$ and $38''$ from the centre, they have radial velocities relative to the cluster of –36.7 km s^{-1} and +32.4 km s^{-1}, corresponding to 4.0 and 3.6 times the core velocity dispersion of $\sigma_p(\mathrm{core}) = 9.1$ km s^{-1}. The 1.5-yr time baseline during repeated observations and the constancy of the radial velocity values indicate that neither of these two stars is a binary or a pulsating star. Unfortunately, the relatively low mean radial velocity of 47 Tucanae ($V_r \simeq -19$ km s^{-1}) does not allow an immediate discrimination between field stars and members of the cluster. But the positions of these two stars in the color-magnitude diagram and the rather high galactic latitude of 47 Tucanae (b = –44°) both argue for membership. The simplest way to eliminate the remaining tiny doubt about membership of the two stars is to obtain high-resolution spectroscopic observations and deduce the luminosity classes of these two objects.

A plausible mechanism to explain these interlopers is ejection from the core by the recoil from an encounter between a single star and a binary, or between two binary stars. One problem with the ejection mechanism is that most of the stars ejected will be moving across our line of sight and so will not be noticed. Thus even a few observed high-velocity stars imply an uncomfortably large population of stars on radial orbits (Sigurdsson 1991, Phinney & Sigurdsson 1991). A possible solution is to have the encounter create a giant and Davies et al. (1993) have studied this in detail by simulating encounters involving a neutron star and a tidal-capture binary, the latter consisting of a white dwarf and a main-sequence star. Most exchange encounters produced a single merged object with the white dwarf and neutron star engulfed in a common envelope of gas donated by the main-sequence primary of the original binary. But a small fraction of the exchanges caused a merger of the white dwarf and the main-sequence star, with this object (presumably a giant) and the neutron star being unbound and having large relative velocities at infinity.

Radial velocity observations of 548 stars within two core radii of the centre of 47 Tucanae by Gebhardt et al. (1995) have increased the sample of high-velocity stars from 2 to 8, although decreasing their fraction (2/50 in Meylan et al. 1991, and 8/548 in Gebhardt et al. 1995). With velocities more than 32 km/s from the cluster mean (about three times the core velocity dispersion of $\simeq 10$ km/s), these stars are moving at close to the cluster escape velocity. Such velocities raise the question of membership, but the Galaxy model by Bahcall & Soneira (1981) shows that the probability of these stars being foreground halo objects is very low. While this new larger sample shows a lower frequency of high-velocity stars, theoretical studies (e.g., Phinney & Sigurdsson 1991) have demonstrated that the number of high-velocity stars is a powerful tool for probing conditions in the core and testing the dynamical models. However, larger samples are needed to exploit this tool. Such high-velocity stars should also be detected through proper motion studies. A long-term HST program for precise astrometry in the core of 47 Tucanae has started and will provide a complete census of high-velocity stars from their proper motions (Meylan et al. 1996).

X-ray sources. Some of the X-ray sources in globular clusters are among the brightest ones in the sky and were easily discovered with the earliest X-ray facilities. More recently, the highly sensitive X-ray satellites EINSTEIN and especially ROSAT allowed the study of less luminous sources in globular clusters (see, for reviews, Grindlay 1993, Verbunt 1993, 1996a,b, and Bailyn 1996).

There are twelve bright ($L_X \gtrsim 10^{35}$ erg s^{-1}) X-ray sources observed in the galactic globular clusters. The X-ray bursts seen in most of these sources provide compelling evidence that such sources are neutron stars, rather than black holes, accreting matter from a low-mass ($M < 1\ M_\odot$) companion filling its Roche lobe (Verbunt 1993). They are called low-mass X-ray binaries (LMXBs), in contrast to the high-mass X-ray binaries in which the donor is an O or B star.

Pointed EINSTEIN and ROSAT observations led to the discovery of one such bright X-ray source in thirty globular clusters of M31 (Trinchieri & Fabbiano 1991, Magnier 1994). The most recent additions were detected by ROSAT in NGC 6652 and Terzan 6 (Verbunt et al. 1995). ROSAT High-Resolution-Imager (HRI) positions show that all bright X-ray sources are in, or close to, the core of the host globular cluster (Johnston et al. 1995a).

Two factors point towards a dynamical origin for LMXBs in globular cluster. First, these sources are highly overabundant in globular clusters with respect to the galactic disk, and, second, many of the globular clusters which contain LMXBs have collapsed cores with high stellar densities. Consequently, tidal captures (Clark 1975, Fabian et al. 1975) and encounters between a binary and a neutron star (Hills 1976), have been invoked to explain the origin of LMXBs, two dynamical mechanisms which do not operate in the lower stellar density of the galactic disk (see Verbunt 1993 for formation and evolution scenarios).

However, the two known orbital periods of globular cluster LMXBs point towards the fact that the companion of the neutron stars may not be a main-sequence star. The first LMXB is 4U 1820–30, located in NGC 6624,which has an orbital period of about 11 minutes (Stella et al. 1987). Although the variability is observed only in X-ray, King et al. (1993) have found an UV and visible counterpart of 4U 1820–30. The Roche-lobe-filling companion can only be a white dwarf, although direct captures of white dwarfs are unlikely given their small cross section. The collision between a neutron star and a red giant, which would lose its envelope and leave its bare core, may be a solution (Verbunt 1987). The second LMXB is AC211 in M15. It has an orbital period of about 17 hours both in X-ray and optical wavelengths (Ilovaisky et al. 1993). In this case the secondary must be a subgiant rather than a main-sequence star, given its high optical luminosity. The above two cases show that the simple model of LMXBs, made of a neutron star and a main-sequence star, may need to be refined.

The same dynamical process which, in globular clusters, creates the excess of accreting neutron-star binaries with respect to the field should also produce large numbers of accreting white-dwarf binaries, called cataclysmic variables (CVs) (see, e.g., Di Stefano & Rappaport 1994, Livio 1996b). Cataclysmic variables in the field are known to be X-ray sources.

Observations with the ROSAT HRI have resolved the core of several galactic globular clusters. They show multiple faint sources in, e.g., NGC 6397, NGC 6752, and 47 Tucanae (Cool et al. 1993, Johnston et al. 1994, Hasinger et al. 1994). A total of about 30 dim sources, either single or multiple, have been detected in or close to the cores of 18 galactic globular clusters, with luminosities in the range $10^{31} \lesssim L_X \lesssim 10^{34}\ \mathrm{erg\,s^{-1}}$ (Johnston & Verbunt 1996).

Repeated observations of the core of 47 Tucanae show that the dim sources are highly variable. In Fig. 9.10, of the four sources detected in April 1992, only two are again detected in April 1993, together with one new source (Hasinger et al. 1994), and one of the sources missing in April 1993 appears again in December 1994 (Verbunt 1996b). The absolute positional accuracy

of the ROSAT HRI is about $5''$, a value which precludes certain identification of any of the ROSAT sources with either the single EINSTEIN X-ray source (Hertz & Grindlay 1983) or with any of the UV variable stars (Aurière et al. 1989 from the ground; Paresce et al. 1992, Paresce & De Marchi 1994, and Meylan et al. 1996 with HST).

Fig. 9.10. Three different ROSAT HRI observations of the core of 47 Tucanae, separated by more than one year. The area is $100'' \times 100''$. All X-ray detected photons are displayed; circles in the left and middle panels encircle photons of the four sources detected in the first observation (left panel) (from Verbunt 1996b Fig. 3).

Contrary to the bright ($L_X \gtrsim 10^{35}$ erg s^{-1}) X-ray sources, the real nature of the low-luminosity ($L_X \lesssim 10^{35}$ erg s^{-1}) X-ray sources, first observed with the EINSTEIN satellite by Hertz & Grindlay (1983), is still unknown. They have been suggested to be cataclysmic variables by Hertz & Grindlay (1983) and Grindlay et al. (1984). Unfortunately, unambiguous detections of CVs in globular clusters have proven to be extremely difficult (see, e.g., Shara et al. 1994, 1995). It is only with the high spatial resolution of HST that candidates have been found: the dwarf nova outburst in 47 Tucanae (Paresce & De Marchi 1994) and the optical counterpart for the historical nova in M80 (Shara & Drissen 1995). Grindlay et al. (1995) report, from observation with HST, the first spectra of three stars, well below the main-sequence turn-off, located near the centre of the dense collapsed globular cluster NGC 6397. These spectra confirm the suspicion from photometry with HST that these three stars may be the long-sought cataclysmic variables in globular clusters. If so, they are likely to be the counterparts of some of the five dim X-ray sources observed by ROSAT in this cluster (Cool et al. 1993, 1995, Grindlay et al. 1995).

However, in addition to the possibility of being CVs, and partly on the basis of their luminosity distribution, it has also been suggested that the low-luminosity X-ray sources are (i) soft X-ray transients, i.e., neutron stars accreting mass from a companion, but at a low rate (Verbunt et al. 1984), (ii) conglomerates of RS CVn binaries or single binaries (Bailyn et al. 1990, Verbunt

et al. 1993), or (iii) radio pulsars (Verbunt & Johnston 1996).

Fig. 9.11 displays the X-ray luminosity distributions (L_X between 0.5 and 2.5 keV) for chromospherically active binaries (RS CVn), non-magnetic cataclysmic variables (CV), recycled millisecond radio pulsars (ms PSR), soft X-ray transients in the galactic disk (SXT), and dim X-ray sources in globular clusters (Glob. Cl.) (Verbunt et al. 1994 and Johnston et al. 1995b). It is conspicuous that the distribution of the dim X-ray sources in globular clusters (Glob. Cl.) overlaps with the distribution of soft X-ray transients in the galactic disk (SXT) and with the bright end of the distribution of cataclysmic variables in the galactic disk (CV). The fact that some of the dim X-ray sources in globular clusters, which were previously detected as single sources, appear now to be multiple, has been taken into account (Verbunt 1996b). E.g., the sources found by Hasinger et al. (1994) in the core of 47 Tucanae (see Fig. 9.10) have luminosities $L_X \gtrsim 10^{33}$ erg s^{-1}, i.e., higher than those observed with ROSAT for cataclysmic variables but compatible with those observed for soft X-ray transients in quiescence. Given the status of our present knowledge, it is reasonable to think that the dim X-ray sources in globular clusters with $L_X \gtrsim 10^{33}$ erg s^{-1} may be soft X-ray transients in their low state, while the dim X-ray sources with $L_X \lesssim 10^{33}$ erg s^{-1} may be cataclysmic variables created by dynamical processes in the dense cores of collapsed clusters (see also Livio 1994 and van den Heuvel 1994 for interesting reviews).

Fig. 9.11. X-ray luminosity distributions for chromospherically active binaries (RS CVn), non-magnetic cataclysmic variables in the galactic disk (CV), recycled millisecond radio pulsars (ms PSR), soft X-ray transients in the galactic disk (SXT), and dim X-ray sources in globular clusters (Glob. Cl.) (from Verbunt 1996b Fig. 4)

Pulsars. Since the discovery by Hulse & Taylor (1975) of the first binary radio pulsars (see Taylor 1994 for a review), these astronomical rotational clocks and their companions have been intensively used in fields as different as relativistic

gravity, nuclear equations of state, neutron star magnetospheres and masses, planet formation, and the dynamical evolution of globular clusters (Phinney 1992). See also Phinney (1993), Verbunt (1993), Phinney & Kulkarni (1994), and Phinney (1996) for recent reviews on pulsars in globular clusters.

Most of the roughly 700 pulsars discovered in our galaxy are single neutrons stars. Observational evidence points towards their origin in supernova explosions, by the collapse of the cores of massive stars, with initial masses $M_i \gtrsim M_{core} \simeq 8\ M_\odot$. Such massive stars have not existed in galactic globular clusters for more than 10 Gyr, although pulsars much younger than this age are found in these stellar systems. Observations have unveiled the presence of more than 30 radio pulsars in galactic globular clusters (see Table 1 in Phinney 1996), out of which eight are members of M15 and eleven are members of 47 Tucanae, two high-concentration clusters. The closest galactic analogues to the globular cluster pulsars are the 34 binary pulsars and the 27 millisecond pulsars, which have pulse periods shorter than 10 ms: these have a distribution of pulse period and spin-down rate very different from that of the bulk of field pulsars, and very similar to that for globular cluster pulsars (see Phinney 1996 Fig. 1). The high rate of occurrence of pulsars in globular clusters is conspicuous when it is considered that globular clusters contain only about 0.05% of the mass of the galaxy. Pulsars, in a way similar to LMXBs, originate mostly in high stellar-density environments.

Soon after the discovery by Backer et al. (1982) of the first millisecond binary pulsar in the field, and by McKenna & Lyne (1988) of the first millisecond binary pulsar in a globular cluster, it was suggested that they resulted from the spin-up of an old neutron star by accretion of matter from a companion star as it overflowed its Roche lobe during its giant phase (Smarr & Blandford 1976). It is the same process witnessed in low-mass X-ray binaries, and parallels have been drawn between (i) high-mass X-ray binaries and high-mass binary radio pulsars, and (ii) low-mass X-ray binaries and low-mass binary radio pulsars. See, e.g., Table 1 in Verbunt (1993) for similar properties between members of these four families (Kulkarni et al. 1990; see Phinney & Kulkarni 1994, and Lyne 1995 for reviews).

Since potential donors in globular clusters have masses smaller than about 0.8 $M_\odot$, globular cluster pulsars are only of the low-mass type. They are interpreted as old neutron stars or white dwarfs recycled into pulsars by accretion from a companion in a binary system, i.e., they are the descendants of low-mass X-ray binaries. The name "recycled pulsars" designates the members of the class of low magnetic field strength, short period, and frequently binary pulsars. It is worth mentioning that, given the typical distance of a globular cluster, ~ 5 kpc, only the brightest pulsars have been detected. Consequently, any calculations of the rate of formation and total number of recycled pulsars in a given cluster require considerable extrapolation.

The formation and properties of cluster pulsars are inextricably linked to the dynamical histories of their host globular clusters. Our understanding of cluster evolution has recently benefited from the discovery of primordial and newly-formed binaries in globular clusters (Hut et al. 1992a) and from improved

computer simulations (Murphy et al. 1990, Gao et al. 1991, Heggie & Aarseth 1992, and Sigurdsson & Phinney 1995).

Globular clusters must be very efficient in recycling their old pulsars, i.e., neutron stars. There are three types of binaries containing neutron stars: (i) primordial binaries, which survived the supernova explosion (they are the only important source of recycled pulsars in the galactic field; (ii) tidal-capture binaries (made possible because of the high stellar density in globulars) in which a neutron star captured or disrupted a non-degenerate star during a close encounter occurring during a 2-body fly-by or an interaction between a single or binary star with another binary; (iii) exchange binaries (also made possible because of the high stellar density in globulars) in which a neutron star has been substituted for one of the original members of a binary which did not initially contain any neutron stars (Phinney 1996). A majority of globular cluster pulsars are single, at variance with galactic field recycled pulsars, since they may have lost their companions through a variety of scenarios: exchange (Phinney & Sigurdsson 1991, Sigurdsson & Phinney 1993), giant capture (Romani et al. 1987, Rappaport et al. 1989), main sequence collisions (Krolik et al. 1984), and evaporation (Ruderman et al. 1989).

Much of the above description of primordial binaries and tidal captures applies if the neutron stars are replaced by white dwarfs. If the donor transfers enough mass to the accreting white dwarf, bringing it above the Chandrasekhar limit, the latter may transmute into a neutron star by "Accretion Induced Collapse" (Canal et al. 1990, Nomoto & Kondo 1991). One advantage of this scenario is its capability of producing neutron stars with or without recoil velocities and with or without weak magnetic fields. The absence of recoil means that very little mass is lost during the supernova event, allowing all binary systems to survive this delicate phase. This has prompted Bailyn & Grindlay (1990) to suggest "Accretion Induced Collapse" as an efficient way to produce neutron stars and hence pulsars in globular clusters.

The detailed physics of all the formation mechanisms briefly described above is not well understood. Even general points remain unknown. E.g., in the case of "Accretion Induced Collapse", does the white dwarf explode or implode to form a millisecond pulsar? Most models are too vague in their predictions and most observations are too scant to allow meaningful comparison with theory. Recently, the timing measurements of three cluster pulsars show that they are young, single, with a strong magnetic field, and clearly members of galactic globular clusters; all three objects have properties typical of pulsars in the galactic field, and the origin of such apparently young objects in very old stellar systems is not understood (Lyne et al. 1996).

It is worth mentioning that the first of the eight pulsars discovered in M15, viz. PSR 2127+11A, has a period of 0.111 second and provided quite a surprise. It was the first of the 500 known pulsars at that time to have a negative period derivative (Wolszczan et al. 1989). As this pulsar is clearly not in a binary system, its change in period is attributed to the acceleration of the pulsar towards the Earth as it moves through the gravitational potential of the cluster. Another such pulsar, viz. PSR 2127+11D, has been discovered

in M15. Both have $\dot{P}/P = -2 \times 10^{-16}$ s^{-1}, a value which has been used to get otherwise unobtainable information on the density and the masses of the stellar remnants in the core of this globular cluster (Phinney 1992).

10. Late phases of evolution and disruption

10.1 Gravothermal oscillations; post-collapse evolution

At one time, a review of the dynamical evolution of globular star clusters might have ended after §9.1. It was not at all certain that a cluster could survive beyond the end of core collapse, and indeed empirical studies of the distribution of central relaxation times of galactic globular clusters (Lightman et al. 1977) were consistent with the idea that clusters somehow suddenly disappeared. Thus, for a long period in the history of cluster studies many experts doubted whether the study of post-collapse clusters had any relevance to the interpretation of observations. Modelling of M15 (see §§9.2 and 9.3) forced a change of attitude, and now a significant proportion of clusters are interpreted as exhibiting the structural characteristics of post-core collapse evolution (cf. §9.2). Even before this was realised, however, it was already becoming clear that the statistics of core parameters could *not* in fact be understood if it was assumed that the entire present population of galactic globular clusters was still undergoing core collapse (Cohn & Hut 1984).

Because of the role of stellar collisions, and other factors, the theoretical behavior of a star cluster after core collapse is subject to some uncertainty, but by now several simplified models exist. All of them depend on providing a flow of energy from the central parts of the cluster, and they differ essentially only in the main physical mechanism which is assumed to be responsible for this. Several processes have been in favor at one time or another, including different kinds of binaries (primordial, tidal and three-body), a massive central black hole (Shapiro 1977) and mass loss from evolution of merger products. At present, probably the favored mechanism is that provided by primordial binaries, a relatively old idea which was revived in recent times by Goodman & Hut (1989). The main uncertainty is the way in which the dynamical behavior of binaries is affected by finite-size effects. Despite such gross uncertainties, post-collapse evolution is worth studying in some detail because, as pointed out by Hénon (1975), many aspects of the evolution appear to be independent of details of the mechanism of energy generation, and we concentrate on these.

In an *isolated* system the outpouring of energy from one of these mechanisms leads to an overall expansion of the cluster, first modelled by Hénon (1965). Relatively little mass is lost on the expansion time scale, and the size of the system varies nearly as $r_h \propto (t - t_0)^{2/3}$, where t_0 is a constant. This follows from equating the expansion time scale to the half-mass relaxation time

(Eq. 7.2), if the mass is constant; Goodman (1983c) has described models in which this is relaxed slightly. When the cluster is tidally limited, the outpouring of energy drives mass across the tidal boundary, and the half-mass radius decreases to maintain constant mean density (the usual condition for tidally limited stellar systems). Thus $r_h \propto (t_0 - t)^{1/3}$, where t_0 is a different constant, and the system contracts. Mass is lost nearly linearly with time (Hénon 1961).

All these results stem from simple theoretical ideas, and are most easily developed for systems of stars of equal mass. Detailed numerical investigations have been used to explore more realistic models, with suitable assumptions about the mechanism of energy generation, where appropriate. N-body models consisting of point masses of equal mass, in which the mechanism is binary formation by three-body encounters, confirm the post-collapse expansion predicted by simplified models (e.g., Giersz & Heggie 1994). The presence of a spectrum of masses does not appear to complicate the evolution, even though continued mass segregation might be expected to occur. This is shown by recent N-body models (Giersz & Heggie 1996a; see also Inagaki 1986c).

Even when the variety of mechanisms considered by Stodółkiewicz (1985) is included, one still observes the same linear dependence of total mass on time, as predicted by Hénon. Since this result determines the lifetime of a cluster, it is worth recording the numerical value. In the case of equal masses (Hénon) the lifetime equals approximately 22.4 current half-mass relaxation times. For the models of Stodółkiewicz (1982), which have unequal masses and many other features, the corresponding value is about 8.7. There are also considerable differences in the structure. For Hénon's model the ratio of the tidal and half-mass radii is $r_t/r_h \simeq 6.9$, whereas for the models of Stodółkiewicz the corresponding number is nearer 2.4. Similar values (around 2.5) are found in N-body models in post-collapse evolution by Giersz & Heggie (1996b), but such values are greatly at variance with typical observational determinations for post-collapse clusters. For instance Meylan & Mayor (1991) found $r_t/r_h \simeq 12$ for most of their models of NGC 6397. It is possible that this is a manifestation of the dependence of the tidal radius on the orbital phase, because all of the theoretical values assume that the tidal environment is static (except for disk shocking, in the case of the models of Stodółkiewicz).

When it comes to the evolution of the core, the nature of the mechanism for generating energy is all-important. If we assume that there are no primordial binaries and that the post-collapse evolution is powered by binaries formed in three-body interactions, then extremely high core densities are required, just as at the close of core collapse (if this is arrested by three-body binaries). These are circumstances in which the central parts of the cluster may be gravothermally unstable. Now, however, the presence of binaries prevents indefinite collapse of the core, but the emission of energy from their evolution can cause a drop in the temperature of the core, which drives the gravothermal instability in reverse, i.e., it drives an expansion of the core. What happens next, at least for systems with more than a few thousand stars (Goodman 1987, Heggie & Ramamani 1989, Breeden et al. 1994), is a complicated succession of collapses and expansions, called "gravothermal oscillations" by their discover-

ers (Sugimoto & Bettwieser 1983, Bettwieser & Sugimoto 1984; see also Fall & Malkan 1978 for a curious precursor of this discovery, and Heggie 1994 for a recent review). The oscillations are superimposed on an overall expansion which approximately follows simple theoretical relationships such as those summarised above (Bettwieser & Fritze 1984). After deep core collapse, however, the early expansion should follow a somewhat different scaling described by Inagaki & Lynden-Bell (1983). Earlier studies of post-collapse evolution (Heggie 1984, 1985) missed the oscillations for numerical reasons.

Fig. 10.1. Core collapse in systems with equal masses (from Makino 1996b Fig. 1). The logarithm of the central density is plotted against time, scaled in proportion to the initial half-mass relaxation time. The successive curves, which correspond to different values of N, have been displaced vertically for clarity.

Quite apart from their relevance in nature (see below), these oscillations are interesting in their own right, as an example of chaotic dynamics. From this point of view they have been studied by Allen & Heggie (1992), Breeden & Packard (1994), and Breeden & Cohn (1995).

Whether these investigations imply that such oscillations should occur in nature is not clear, for a variety of reasons. For several years after their discovery, the oscillations were studied almost entirely with the aid of simplified models, i.e., gas models and Fokker-Planck models (e.g., Hut et al. 1989, Cohn et al. 1989, Spurzem & Louis 1993), and it has been argued that the subtle thermal effects which are responsible are masked, in real systems, by fluctuations (Inagaki 1986b, 1988). Nevertheless, growing evidence from N-body simulations was already pointing in the opposite direction (Bettwieser & Sugimoto 1985, Makino et al. 1986, Makino & Sugimoto 1987, Heggie 1989, Makino 1989, Heggie et al. 1994). In 1995 the genuine occurrence of gravothermal oscillations in N-body systems was spectacularly demonstrated by Makino (Makino 1996a,b; see Fig. 10.1). These results confirm that the nature of post-collapse evolution in N-body systems is far more stochastic than in the simplified continuum models on which so much of our understanding rests at present. It has been known for a long time that the formation and evolution of individual binaries in small N-body systems makes the evolution of the core quite erratic after core collapse (e.g., Sugimoto 1985, McMillan 1986b), and one might have thought that the effects of individual binaries would have been of less significance in much larger systems. But now it is known that gravothermal oscillations make the evolution of such large systems equally erratic. Indeed the interaction between these two processes had already been studied by Takahashi & Inagaki (1991), in a paper which develops an earlier model by Inagaki & Hut (1988); cf. also Spurzem & Giersz (1996).

Even with simplified models it is known that the oscillations tend to be suppressed by the presence of a mass spectrum (Murphy et al. 1990; see also Bettwieser 1985a). The main source of doubt about the significance of these oscillations, however, is concerned with the mechanism of energy generation. Though oscillations also occur if this is caused by tidal-capture binaries (Cohn et al. 1986, Cohn 1988), they may be suppressed by the presence of primordial binaries. These have the effect of preventing the phases of extremely high central density which are necessary in well developed oscillations, just as collapse of the core is ended at much lower densities if primordial binaries are present (McMillan et al. 1990, 1991, Heggie & Aarseth 1992). On the other hand, the steady exhaustion of primordial binaries as an energy source, caused by their destruction in mutual interactions, gradually erodes their effectiveness. The Fokker-Planck models of Gao et al. (1991) suggest that gravothermal oscillations do eventually occur even if the initial abundance of primordial binaries is as high as 20%. These models are approximate in some important ways, however, and do not include a spectrum of masses, but the possibility that clusters with primordial binaries may exhibit oscillations late in the post-collapse phase cannot be ruled out. On the other hand, if the post-collapse evolution is assumed to be steady, the results of Vesperini & Chernoff (1994) can be used to estimate the likely size of the core. The general theoretical issues involved in core size are considered by Hut (1996b).

Whether or not post-collapse oscillations occur is not simply an academic question, as it is directly related to the observable structure of a post-collapse

cluster, especially with regard to the presence or absence of a resolved core. If oscillations occur, then the cluster is likely to be observed close to an expansion phase, when the core may be large enough to be resolved, whereas a much smaller core is expected if the post-collapse evolution is steady (Bettwieser 1985b, Grabhorn et al. 1992). These questions are also involved in the interpretation of the statistics of core parameters.

As already mentioned, another possible mechanism for powering post-collapse expansion is runaway coalescence by two-body interactions (see Lee 1987b). As Goodman (1989) pointed out, the simplest resulting scenario requires a core luminosity which is quite inconsistent with observations, and some more elaborate scenario, such as one involving gravothermal oscillations, is required.

10.2 Disruption

Globular clusters are subject to several disruptive processes, both internal and external. In fact this distinction is not quite clear-cut, as the rate of escape by evaporation depends on the tidal field. Still, among the main internal disruptive processes we include evaporation, either by two-body interactions or those involving binaries (cf. §7.3). A second internal process, of importance mainly for young clusters, is mass loss from stellar evolution (cf. §5.5). External influences include time-dependent tidal fields, among which are disk and bulge shocking (cf. §7.4), and interactions with giant molecular clouds (for which a main reference is still the classic paper Spitzer 1957). Another external destructive mechanism is dynamical friction, which acts on the entire cluster as it ploughs through the Galaxy (Tremaine et al. 1975, Tremaine 1976). Not all mechanisms affecting the population of galactic globular clusters are destructive: successive capture from satellite galaxies may well be a complicating factor (e.g., van den Bergh 1993a,b,c, Fusi Pecci et al. 1995).

Many papers (in the sections which are referred to in the above paragraph) tell us about the effects of these processes on a single cluster, but much is to be learned by analysing the way in which their effect alters the system of galactic globular clusters as a whole. The aim of Fall & Rees (1977; cf. also Fall & Rees 1988) was to show that the range of cluster masses could be accounted for, given a suitable initial correlation of cluster mass and density, if the population evolved by cluster-cluster interactions, disk shocking and (internal) evaporation. Dynamical friction was, they concluded, relatively unimportant, though its efficiency depends on the nature of the galactic potential (Pesce et al. 1992) and it will be important for massive clusters at small radii (Surdin 1978, Capriotti et al. 1996). The effect of mass loss by internal stellar evolution was not considered until relatively recently (cf. §5.5), but was included (with several other processes) in the work of Chernoff & Shapiro (1987). They studied the effect of these processes by assuming that the evolution of individual clusters took place along the King sequence. It has recently been found (Fukushige

& Heggie 1995) that previous estimates of the lifetimes of globular clusters, which had been based on Fokker-Planck modelling, may be underestimates. The error may be as much as a factor of ten in the case of systems which are destroyed quickly.

Fig. 10.2. The effect of several destructive mechanisms on the distribution of galactic globular clusters (from Gnedin & Ostriker 1996 Fig. 20)a. Those clusters for which, at the stated galactocentric radius, the combined theoretical destruction time scale exceeds a Hubble time, are predicted to lie within the corresponding curve. Data for 119 galactic globular clusters are plotted, with symbols determined by the mean galactocentric distance of each cluster, which in turn was estimated according to a simple kinematical model of the cluster system ("OC isotropic"). The other labels indicate the main destructive mechanisms in each domain of the diagram.

Aguilar et al. (1988) gave a more detailed assessment of several of these processes, including also shocks due to the galactic bulge, but concentrated more on determining their *current* effect on the population of galactic globular clusters. They also showed how the relative effectiveness of the processes they considered depended on the orbital characteristics of the clusters. Okazaki &

Tosa (1995) have considered the influence of three of the main processes of dissolution on the *luminosity* function, and it would be interesting to study the "fundamental plane" of cluster properties (Djorgovski 1995, cf. also §4.6 of this review, van den Bergh 1994, Covino & Pasinetti and Fracassini 1993) from this point of view. Using the minimum of theory, Hut & Djorgovski (1992) have estimated that, in our Galaxy, globular clusters are dying at a rate of about 5 per Gyr. This is not dissimilar to the theoretical prediction of Gnedin & Ostriker (1996) that more than half of the present population may disappear within the next Hubble time (Fig. 10.2).

Provided that the effects of these known destruction mechanisms are well understood, they can be used to make inferences about galactic structure. For example the likely effects of a galactic bar were studied by Long et al. (1992), while Surdin (1993) has shown how studies of the galactic globular cluster system from the point of view of disk shocking might be used to constrain the structure of the disk. More speculative destructive mechanisms which could be indirectly studied in this way include hypothetical massive black holes (Wielen 1987, Moore 1993, Charlton & Laguna 1995, Klessen & Burkert 1996). There are other reasons why the study of the dynamical evolution of globular cluster systems may have far-reaching implications. For example, provided that the various mechanisms are well enough understood, they can be applied to external cluster systems, and may well help to explain the variation of the specific cluster frequency with the mass of the parent galaxy (Murali & Weinberg 1996).

11. Future directions

In the realm of modelling, major advances may be expected in the next few years. While Fokker-Planck models will continue to provide much information on problems of interest, an increasing role will be played by N-body methods. At present these suffer from two major deficiencies, as already mentioned in §§8.1 and 9.5, i.e. (i) the fact that N is still much too small, and (ii) the absence of an adequate treatment of stellar collisions.

The first problem will *eventually* be solved by advances in computer speed. Unfortunately, the computational effort (Hut et al. 1988) grows with N roughly as N^α with $2 < \alpha < 3$. If we suppose that computing speed roughly doubles each year, then it is clear that the step from the largest simulation which is feasible at present on a general-purpose computer ($N \sim 10^4$, Spurzem & Aarseth 1996 – with great effort!) to a sizeable globular cluster ($N \sim 10^6$) could not be taken within the next decade. The development of special-purpose hardware, however, is transforming the picture. The GRAPE/HARP project, successfully developed at the University of Tokyo over the last few years (Makino et al. 1993, Makino 1996a,b) now provides the ability to model systems of at least 3×10^4 stars in a reasonable time. Within five years it would, in principle, be straightforward to build a hardware which would increase this by another order of magnitude.

Striking as such advances are, little can be done with a single model; some further time will elapse before the computation of such models becomes routine, and this is necessary if it is desired to investigate the effects of different parameters on the evolution. Therefore there can be no doubt that the other simpler methods mentioned in §8 will continue to provide much of our detailed information on the evolution of star clusters for some years to come. Here one of the most promising developments is in Fokker-Planck models which incorporate aspects of Monte Carlo methods. Such a method was already used over 10 years ago by Stodółkiewicz (1985) to produce some of the most realistic models of globular clusters that have yet been published, and they included an astonishingly wide range of physical processes. Their chief restriction was in the small number of "stars" that could be handled at that time, but the subsequent ten years of developments in general-purpose hardware should make possible a dramatic improvement (Giersz, pers. comm.) The cost of increasing the number of stars by a given factor is considerably smaller than in direct N-body models, and there are several reasons why it is important to use larger N. For example, it is difficult to study the evolutionary effects of rare species (e.g., stellar-mass black holes), because none may be present in a scaled-down model!

The other main obstacle to progress, within the context of N-body models, is the handling of non-point mass effects. Already much could be done with existing codes, in which the outcome of a collision is determined by a simple prescription, such as might be suggested by results of simulations using smooth particle hydrodynamics (§9.4). The next step is to incorporate the SPH within an N-body code, so that the simple prescription is replaced by a detailed modelling of the particular collision that is occurring. Indeed small-scale test calculations of this kind have been carried out (McMillan, pers. comm.). Greater difficulties will occur in modelling interactions between stars in binary systems, at the point where they exchange mass over long periods of time, and in incorporating the effects of stellar evolution on this and other processes (Livio 1996a, Zwart 1996, Hut 1996a). Aarseth (1996a,b) is making great progress here.

There are essential points at which observations are needed to supply more reliable parameters for the N-body models. In relation to the primordial binaries, what is the distribution of the masses of the components, and that of the semi-major axes? These are significant factors in determining how effective the binaries can be at powering the evolution of a cluster for its entire life, from birth to dissolution. What is the present fraction of binaries? What is their spatial distribution? Though much has been learned from radial velocities of giants (§9.6), thanks to the advent of multiple-fiber devices, it is just now becoming possible to extend the statistics to the upper main sequence (Côté & Fischer 1996), where binaries of shorter periods will become observable.

The same multiple-fiber devices have provided, during these last few years, increasingly large samples of stellar radial velocities, but the next important improvement should come from the observation of proper motions, whose large potential of dynamical information has not been exploited yet. E.g., the

data obtained in ω Centauri (Reijns et al. 1993 and Seitzer, pers. comm.) — proper motions for about 7,000 stars and stellar radial velocities for about 3,500 stars — will permit investigation of the 3-D space velocity distribution and rotation. The same spectra are also being used to determine metallicity, to investigate the correlation between metallicity, radius, and kinematics. A quantum jump in the understanding of the internal dynamics of this globular cluster will result from the interpretation of these data.

Extensive and deep multicolor imaging should be obtained at different radii from the centre of the clusters, in order to provide a clear and more detailed view on the influence of dynamics on stellar evolution, from color-magnitude diagrams and color gradients.

An essential parameter to be supplied for the N-body models concerns the luminosity and mass functions, and especially their lower parts, possibly lower ends. E.g., in the case of NGC 6397, proper motions from HST should soon provide the first clear observation of a globular mass function close to the hydrogen-burning limit by allowing a clear distinction between cluster members and field stars (King, pers. comm.).

The above few points illustrate that theorists are gradually turning from the rather "pure" types of stellar dynamical calculations, which have tended to dominate the subject in recent decades, to more realistic simulations, e.g., from equal-mass systems to those with a mass spectrum, and from isolated systems to ones which are tidally truncated. The same is true for observers who, as the instruments improved, have gradually abandoned the idealised vision of a dormant swarm of stars, whose members were thought to evolve individually. Not only are the theoretical dynamical simulations gradually becoming more and more realistic, but they are increasingly directed to the questions posed by observations, e.g., the influence of dynamics on the mass spectrum.

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